SECTION A

QUESTION 1

(a)

(1) $(x-1)^2 = 2(1-x)$

$\therefore x^2 - 2x + 1 = 2 - 2x$

$\therefore x^2 - 1 = 0$

$\therefore (x-1)(x+1) = 0$

$\therefore x = 1 \quad \text{or} \quad x = -1$ (4)

(2) $5^{-x}.5^{x-2} = \dfrac{25^{2x}}{5}$

$\therefore 5^{-x+x-2} = \dfrac{\left(5^2\right)^{2x}}{5^1}$

$\therefore 5^{-x+x-2} = 5^{4x-1}$

$\therefore -x + x - 2 = 4x - 1$

$\therefore -1 = 4x$

$\therefore -\dfrac{1}{4} = x$

$\therefore x = -\dfrac{1}{4}$ (4)

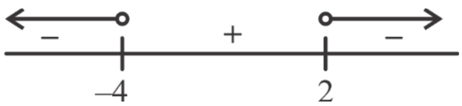
(b) $(x+1)^2 < 9$

$\therefore x^2 + 2x + 1 - 9 < 0$

$\therefore x^2 + 2x - 8 < 0$

$\therefore (x+4)(x-2) < 0$

Critical values: $x = -4$ or $x = 2$



$\therefore -4 < x < 2$ (4)

(c) If 2 and –4 are roots of the equations $(x-2)$ and $(x+4)$ are factors

$\therefore (x-2)(x+4) = 0$

$\therefore x^2 + 2x - 8 = 0$

$\therefore b = 2 \quad \text{and} \quad c = -8$ (3)

(d)

(1) $y = \frac{-4}{y} - 4$

$\therefore y^2 = -4 - 4y$

$\therefore y^2 + 4y + 4 = 0$ (2)

(2) $\Delta = b^2 - 4ac$

$= 4^2 - 4(1)(4)$

$= 0$

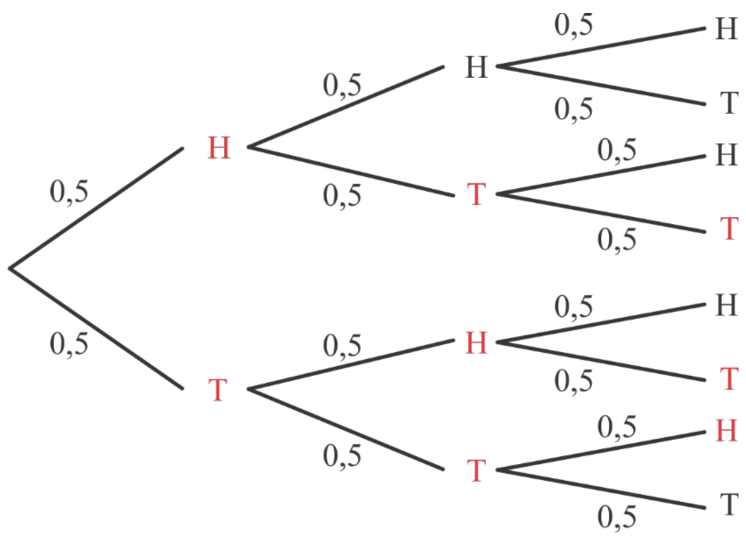
$\therefore$ Roots are real and equal (2)

[19]

QUESTION 2

(a)

(1)



(3)

(2) The events are: $E = \{HTT; THT; TTH\}$

$\therefore P(2 \text{ tails and } 1 \text{ head}) = \frac{3}{8}$ (2)

(b)

(1) $P(A \cap B) = 0$ (1)

(2)

(i) You cannot pick a R2 and a R5 coin at the same time. (1)

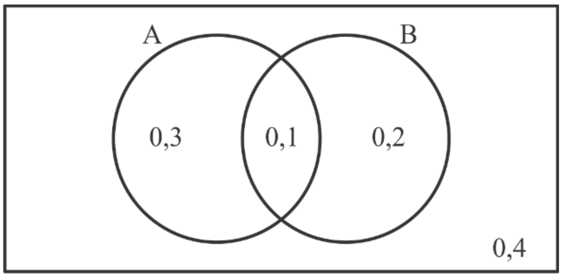
(ii) $P(\text{either a R5 or a R2}) = P(\text{R5}) + P(\text{R2}) - P(\text{R5 and R2})$

$= 0{,}36 + 0{,}47 - 0$

$= 0{,}83$ (3)

(c)

(1)



(4)

(2) $\text{P}\left(\text{exactly one machine is stamping R5 coins}\right) = 0,3 + 0,2$

$= 0,5$ (3)

[17]

QUESTION 3

(a) $480163 \div 0,502 = \text{R}956\,500$ (2)

(b) $\text{R}956\,500 \times 5\% = \text{R}47\,825$ (2)

(c) $\text{Total cost including import charges} = 956\,500 + 47\,825$

$= \text{R}1\,004\,325$

$$\text{A} = P\left(1+i\right)^n$$

$$\therefore 1\,004\,325 = 225\,450\left(1+0,095\right)^n$$

$$\therefore \frac{1\,004\,325}{225\,450} = \left(1+0,095\right)^n$$

$$\therefore n = \log_{1+0,095}\left(\frac{1\,004\,325}{225\,450}\right)$$

$$= 16,46$$

$$= 17 \text{ years}$$ (4)

(d)

(1) $\text{Loan amount} = 1\,004\,325 - 225\,450$

$= \text{R}778\,875$

$$\text{P} = \frac{x\left[1-\left(1+i\right)^{-n}\right]}{i}$$

$$\therefore 778\,875 = \frac{x\left[1-\left(1+\frac{0,12}{12}\right)^{-(4\times 12)}\right]}{\frac{0,12}{12}}$$

$$\therefore x = 778\,875 \div \frac{\left[1-\left(1+\frac{0,12}{12}\right)^{-(4\times 12)}\right]}{\frac{0,12}{12}}$$

$$= 20\,510,76607$$

$$= \text{R}20\,510,77$$ (4)

(2) **Method 1:** Outstanding balance $= \text{A} - \text{F}$

$$\text{OB} = 778875\left(1+\frac{0,12}{12}\right)^{24} - \frac{20510,76607\left[\left(1+\frac{0,12}{12}\right)^{24}-1\right]}{\frac{0,12}{12}}$$

$$= \text{R}435718,15$$

Method 2: Using Present value with *n* being the number of payments left

$$\text{OB} = \frac{20510,76607\left[1-\left(1+\frac{0,12}{12}\right)^{-24}\right]}{\frac{0,12}{12}}$$

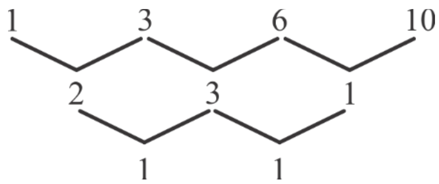
$$= \text{R}435718,15$$ (3)

[15]

QUESTION 4

(a)

(1) Constant 2nd difference = 1. Therefore, the sequence is quadratic. (1)



(2) $T_n = an^2 + bn + c$

$2a = 1$ $\quad$ $3a + b = 2$ $\quad$ $a + b + c = 1$

$\therefore a = \frac{1}{2}$ $\quad$ $\therefore 3\left(\frac{1}{2}\right) + b = 2$ $\quad$ $\therefore \frac{1}{2} + \frac{1}{2} + c = 1$

$\therefore b = \frac{1}{2}$ $\quad$ $\therefore c = 0$

$$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n$$ (6)

(b) $T_3 = 52$ and $T_7 = 78$

$T_3 = a + 2d$ $T_7 = a + 6d$

$\therefore 52 = a + 2d \rightarrow \boxed{1}$ $78 = a + 6d \rightarrow \boxed{2}$

From $\boxed{2} - \boxed{1}$:

$\therefore 4d = 26$

$\therefore d = \frac{13}{2} \rightarrow$ sub into $\boxed{1}$

$\therefore 52 = a + 2\left(\frac{13}{2}\right)$

$\therefore a = 39$

$T_{43} = 39 + 42\left(\frac{13}{2}\right)$

$= 312\,\text{cm}$ (5)

[12]

QUESTION 5

(a)

(1) $f(x) = x^2 - 6x + 9$ and $f(x+h) = (x+h)^2 - 6(x+h) + 9$

$= x^2 + 2xh + h^2 - 6x - 6h + 9$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h + 9 - \left(x^2 - 6x + 9\right)}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 6h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}}$$

$= 2x - 6$ (5)

(2) $f'(-3) = 2(-3) - 6$

$= -12$ (2)

(3) $y = \pi x^{-1} + 3x^{\frac{1}{3}}$

$$\frac{dy}{dx} = -\pi x^{-2} + 3\left(\frac{1}{3}\right)x^{-\frac{2}{3}}$$

$= \dfrac{-\pi}{x^2} + \dfrac{1}{x^{\frac{2}{3}}}$ (4)

[11]

$\boxed{\textbf{74 Marks}}$

SECTION B

QUESTION 6

(a)

(1) Domain: $x \in \mathbb{R}; x \neq 3$ (1)

(2) Range: $y \in \mathbb{R}; y \neq -3$ (1)

(3)

(i) 5 units (1)

(ii) 5 units (1)

(b)

(1) $y = a.b^x \rightarrow$ substitute $\left(0;\frac{1}{4}\right)$

$\therefore \frac{1}{4} = a.b^0$

$\therefore a = \frac{1}{4}$

$\therefore y = \frac{1}{4}(b)^x \rightarrow$ substitute $\left(2;\frac{9}{4}\right)$

$\therefore \frac{9}{4} = \frac{1}{4}(b)^2$

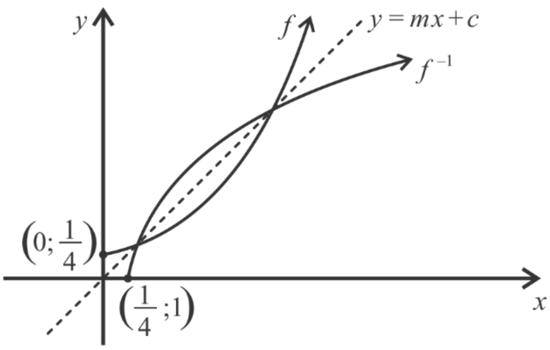
$\therefore b^2 = 9$

$\therefore b = \pm 3$

but $b > 0$

$\therefore b = 3$ (4)

(2)



Marks are awarded for:
Correct shape of f ✓
Correct domain of f ✓
Correct y-intercept of f ✓
Correct shape of f^{-1} ✓
Correct range of f^{-1} ✓
Correct x-intercept of f^{-1} ✓

(3)

(3) Range: $y \in \left[\frac{1}{4};\infty\right)$ (1)

(4) $f: \quad y = \frac{1}{4}(3)^x$ where $x \geq 0$ and $y \geq \frac{1}{4}$

$f^{-1}: \quad x = \frac{1}{4}(3)^y$

$\therefore 4x = 3^y$

$\therefore y = \log_3 4x$ where $y \geq 0$ and $x \geq \frac{1}{4}$ (3)

(5) Shown on the graph for question 6b number 2 above. (3)

[18]

QUESTION 7

(a) $f(x) = x^2 + 6x + (3)^2 + 5 - (3)^2$

$= (x+3)^2 - 4$

$\therefore \text{TP}(-3;-4)$ (3)

(b)

(1) $x^2 + 6x + 5 = -x - 5$

$\therefore x^2 + 7x + 10 = 0$

$\therefore (x+5)(x+2) = 0$

$\therefore x = -5 \quad \text{or} \quad x = -2$

$\therefore \text{A}(-5;0) \quad \text{and} \quad \text{B}(-2;-3)$ (4)

(2) Horizontal shift: $-5 < t < -2$ (3)

(c)

(1) $\text{MN} = (-x-5) - (x^2 + 6x + 5)$

$= -x - 5 - x^2 - 6x - 5$

$= -x^2 - 7x - 10$

For Max length: $\text{MN}'(x) = 0$

$\text{MN}'(x) = -2x - 7$

$\therefore -2x - 7 = 0$

$\therefore x = -\frac{7}{2}$

$\therefore$ Max length of MN $= -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10$

$= \frac{9}{4}$ units (6)

(2) $k > \frac{9}{4}$ (1)

[17]

QUESTION 8

(a)

(1) If $x = -\frac{3}{2}$ the series is: $\frac{3}{2}; -\frac{9}{2}; \frac{27}{2}; \ldots$

$\therefore$ The series is geometric but $r = -3$ and it will not converge. (3)

(2) $\dfrac{x-3}{x+3}=\dfrac{12-x}{x-3}$

$\therefore (x-3)^2=(12-x)(x+3)$

$\therefore x^2-6x+9=12x+36-x^2-3x$

$\therefore 2x^2-15x-27=0$

$x=9 \quad \text{or} \quad x\neq -\dfrac{3}{2}$ (4)

(b) $S_4=\dfrac{15}{2}$; $S_5=\dfrac{31}{2}$ and $S_6=\dfrac{63}{2}$

$T_5=\dfrac{31}{2}-\dfrac{15}{2}\rightarrow S_5-S_4$

$=8$

$T_6=\dfrac{63}{2}-\dfrac{31}{2}\rightarrow S_6-S_5$

$=16$

$r=\dfrac{T_6}{T_5}=\dfrac{16}{8}=2$

$T_5=ar^4$

$\therefore 8=a(2)^4$

$\therefore a=\dfrac{1}{2}$

$S_n=\dfrac{a\left[r^n-1\right]}{r-1}$

$\therefore S_n=\dfrac{\frac{1}{2}\left[(2)^n-1\right]}{2-1}$

$=\dfrac{2^n-1}{2}$ (7)

[14]

QUESTION 9

(a) $f(x)=-x^3+bx^2+cx-3$

$f(1)=4$

$\therefore 4=-(1)^3+b(1)^2+c(1)-3$

$\therefore b+c=8\rightarrow \boxed{1}$

$f'(x)=-3x^2+2bx+c$

$f''(x)=-6x+2b$

$f''\left(\dfrac{1}{2}\right)=1$

$\therefore 1=-6\left(\dfrac{1}{2}\right)+2b$

$\therefore b=2\rightarrow \boxed{2}$

Sub $\boxed{2}$ into $\boxed{1}$:

$\therefore c=6$ and $b=2$ (7)

(b) Concave up: $f''(x) > 0$
$\therefore -6x+4>0$
$\therefore x<\dfrac{2}{3}$ (3)

[10]

QUESTION 10

	Volume	**Rate**	**Time**
Original rate	340	x	$\dfrac{340}{x}$
Increased rate	340	$x+2$	$\dfrac{340}{x+2}$

Original time $-$ faster time $=3$

$\therefore \dfrac{340}{x}-\dfrac{340}{x+2}=2 \rightarrow \text{LCD}=x\left(x+2\right)$

$\therefore 340\left(x+2\right)-340=3x\left(x+2\right)$

$\therefore 3x^2+6x-680=0$

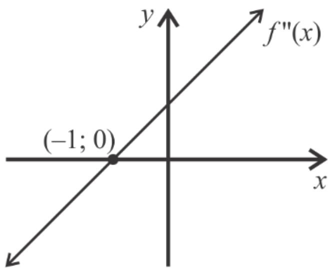
$\therefore x=14,09 \quad \text{or} \quad x\neq -16,09$

$\therefore t=\dfrac{340}{14,09}=24,13\,\text{seconds}$

QUESTION 11

(a)

(1) Horizontal tangent when $f'(x) = 0$: $x=-2$ and $x=0$ (2)

(2) 

(2)

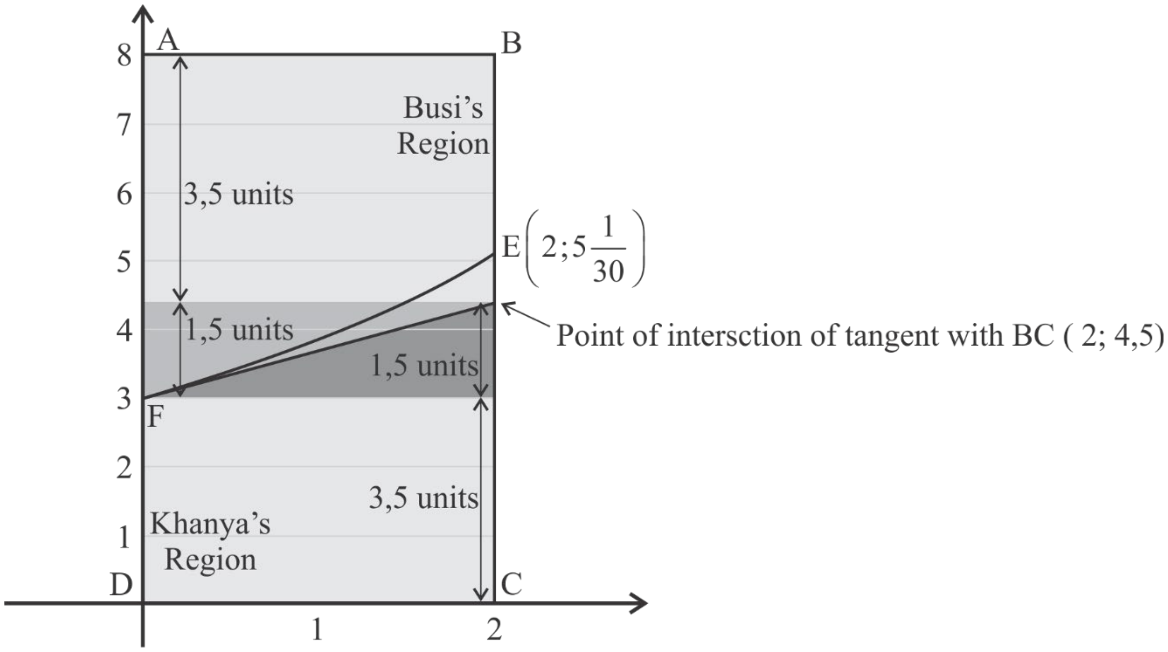
(b) Equation of tangent at F: $y = mx + 3$

$$\frac{dy}{dx} = \frac{3}{15}x^3 + \frac{3}{4} \rightarrow \text{substitute } x = 0$$
$$= \frac{3}{4}$$
$$\therefore m = \frac{3}{4}$$
$$\therefore y = \frac{3}{4}x + 3$$

Substitute $x = 2$ into the equation of the tangent to determine the point of intersection of the tangent with line BC:

$$y = \frac{3}{4}(2) + 3$$
$$= \frac{9}{2}$$

$\therefore$ point of intersection $\left(2;\frac{9}{2}\right) = (2;4,5)$



Area of Khanya's region $= 2\times 3 + \frac{1}{2}(2\times 1,5)$
$= 7,25\,\text{units}^2$

Area of Busi's region $= 2\times 3,5 + \frac{1}{2}(2\times 1,5)$
$= 8,5\,\text{units}^2$

$\therefore$ Busi's Region is larger (7)

[11]

76 Marks

Total: 150 Marks