SECTION A

QUESTION 1

(a)　Solve for *x*:

(1)　$(x-1)^2 = 2(1-x)$　　　　　　　　　　　　　　(4)

(2)　$5^{-x}.5^{x-2} = \dfrac{25^{2x}}{5}$　　　　　　　　　　　　(4)

(b)　Solve for *x* if $(x+1)^2 < 9$.　　　　　　　　　　(4)

(c)　If 2 and –4 are the roots of the equation $x^2 + bx + c = 0$, determine the values of *b* and *c*.　　　　　　　　　　　　　　(3)

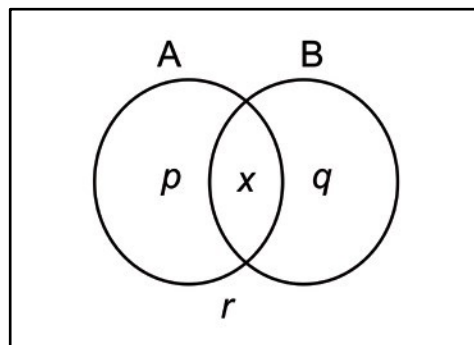(d)　Given: $x - 2 = \dfrac{-4}{x-2} - 4$

If:　　$y = x - 2$

(1)　Show that the given equation can be expressed as $y^2 + 4y + 4 = 0$.　　(2)

(2)　Hence or otherwise show that the equation has real and equal roots.　　(2)

[19]

QUESTION 2

(a)   Three unbiased coins are tossed.

     (1)   Represent all possible outcomes using a tree diagram.   (3)

     (2)   Determine the probability of getting two tails and one head (in any order).   (2)

(b)   (1)   If A and B are mutually exclusive events, write down $P(A \cap B)$.   (1)

     (2)   One coin is randomly picked from a savings jar. The probability of picking a R5 coin out of the savings jar is 0,36 whereas the probability of picking a R2 coin out of the same jar is 0,47.

          (i)   Explain why the events "picking a R5 coin" and "picking a R2 coin" are mutually exclusive events.   (1)

          (ii)   Determine the probability of picking a R2 coin or a R5 coin from the jar.   (3)

(c)   Machine A and machine B are two different coin-pressing machines that operate at the same time. The probability that machine A presses a R5 coin is 0,4 and the probability that machine B presses a R5 coin is 0,3. The probability that the machines both press R5 coins at the same time is 0,1.

     (1)   Redraw the Venn diagram in your Answer Book and fill in numerical values for *p*, *q*, *r* and *x*.

                                                     (4)

     (2)   Calculate the probability that exactly one of the machines is pressing a R5 coin.   (3)

                                                       [17]

QUESTION 3

Round off your answers to 2 decimal digits where necessary, unless stated otherwise.

The owner of a Printing Company has decided to purchase machinery from China.

(a)   The cost of her machinery that will be imported from China is ¥480 163 (i.e. 480 163 Chinese yuan).

If the exchange rate is 1 South African rand = 0,502 Chinese yuan, calculate the total amount she will pay in South African rand.   (2)

(b)   The import charges amount to 5% of the value of the machinery purchased. Calculate the import charges in rand.   (2)

(c)   The owner intends to use her savings to purchase the machinery, which includes the import charges. She currently has R225 450 in her savings account earning interest at 9,5% effective (i.e. 9,5% per annum compounded annually).

Determine **how long** it will take before she has enough money in her savings to purchase the machinery. (Assume that the price of the machinery and the import charges remain constant.)   (4)

(d)   The owner decides that she wishes to purchase the machinery immediately. She **uses her current savings as a deposit** and approaches the bank for a loan for the balance that she requires.

The bank will offer her a loan which must be repaid at the end of each month at an interest rate of 1% per month compounded monthly over a period of 4 years.
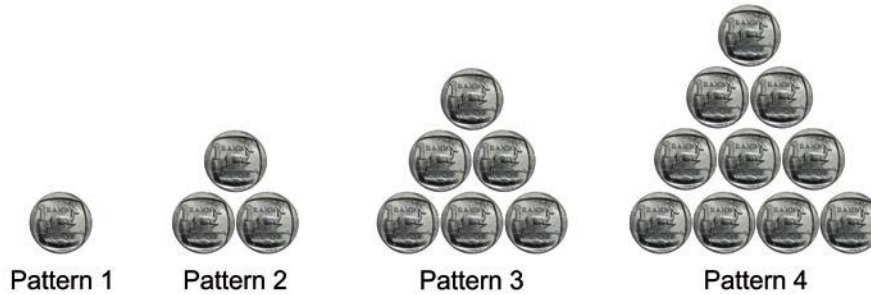
(1)   Calculate the monthly instalment. (Assume she receives the loan immediately and that the first payment is made after one month.)   (4)

(2)   Calculate the outstanding balance at the end of two years, i.e. immediately after the 24th payment.   (3)
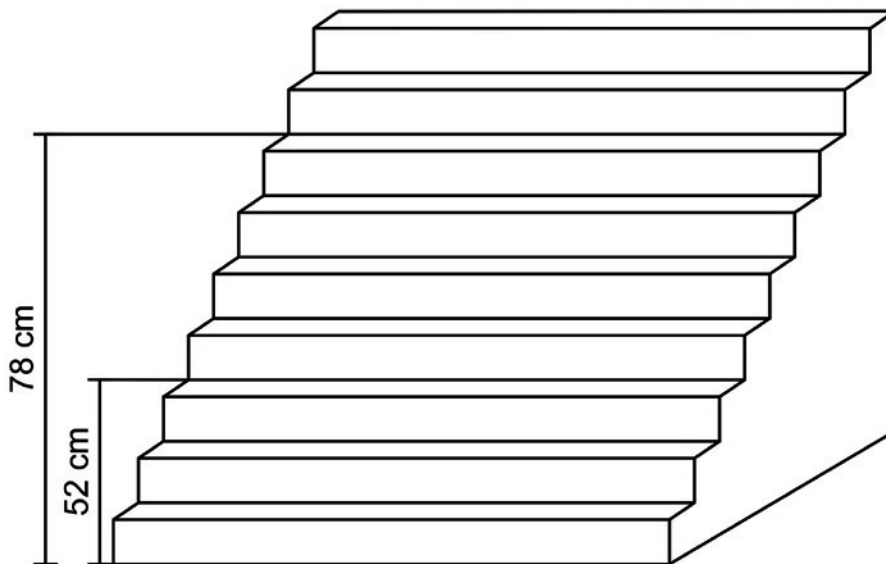
[15]

QUESTION 4

(a)   The first four patterns formed by the arrangement of coins are shown below:

Pattern 1   Pattern 2   Pattern 3   Pattern 4

(1)   Show that the progression of patterns forms a quadratic sequence.   (1)

(2)   Determine the nth term of this quadratic sequence.   (6)

(b)   The heights above ground level of steps in a staircase form an arithmetic sequence.

The heights of the 3rd and 7th steps are 52 cm and 78 cm above the ground respectively.

Determine the height above ground level of the 43rd step.   (5)

[12]

QUESTION 5

(a) $f(x) = (x-3)^2$

   (1)   Determine $f'(x)$ from first principles.   (5)

   (2)   Determine the gradient of f at $x = -3$.   (2)

(b)   Determine $\dfrac{dy}{dx}$: $y = \dfrac{\pi}{x} + 3\sqrt[3]{x}$ (4)

[11]

74 marks

SECTION B

QUESTION 6

(a)   Consider the graphs of $g(x) = \dfrac{-4}{x-3} + 2$ and $h(x) = \dfrac{-4}{x+2} - 3$.

     (1)   Write down the domain of *g*.   (1)

     (2)   Write down the range of *h*.   (1)

     (3)   If the graph of *g* is shifted so that it coincides with the graph of *h*,

          (i)   how many units must the graph of *g* be shifted horizontally?   (1)

          (ii)   how many units must the graph of *g* be shifted vertically?   (1)

(b) $A\left(0; \dfrac{1}{4}\right)$ and $B\left(2; \dfrac{9}{4}\right)$ are two points on the graph of $y = f(x) = a(b)^x$ for $x \geq 0$.

    a and *b* are constants and $b > 0$.

     (1)   Show that $a = \dfrac{1}{4}$ and $b = 3$.   (4)

     (2)   Draw a sketch graph of $f(x) = \dfrac{1}{4}(3)^x$ for $x \geq 0$.

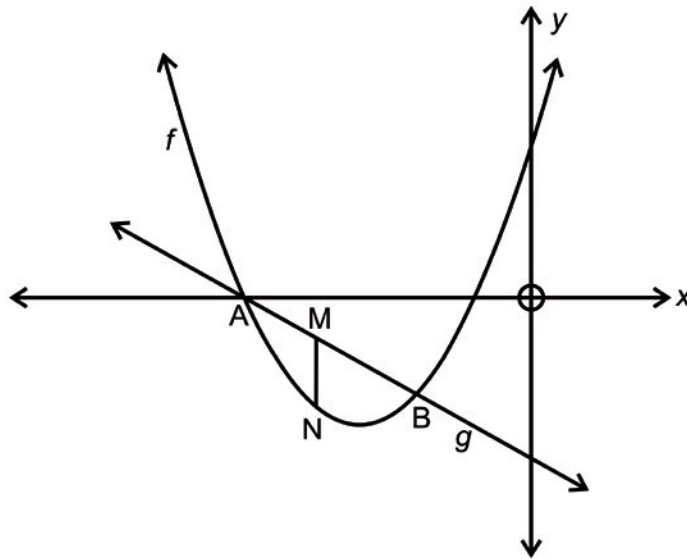   Show the intercepts with the axes.   (3)

     (3)   State the range of *f*.   (1)

     (4)   Determine f^{-1}, the inverse of *f* in the form $y = \ldots$ (3)

     (5)   Sketch the graph of $y = f^{-1}(x)$, on the same set of axes as the graph
          of $y = f(x)$. Show the intercepts with the axes.   (3)

[18]

QUESTION 7

The sketch represents the graphs of the functions $f(x) = x^2 + 6x + 5$ and $g(x) = -x - 5$.

(a)   Rewrite $f(x)$ in the form $f(x) = a(x + p)^2 + q$ and hence write down the turning point of the graph of *f*.   (3)

(b)   (1)   Determine the co-ordinates of A and B, the points of intersection of the two graphs.   (4)

       (2)   Hence, determine the value(s) of *t* for which the equation $(x + t)^2 + 6(x + t) + 5 = -(x + t) - 5$ has one positive and one negative root.   (3)

(c)   (1)   Determine the largest possible value of MN if MN is parallel to the *y*-axis with M a point on the graph of *g* and N a point on the graph of *f*.   (6)

       (2)   Hence, or otherwise, determine the value of *k* for which $f(x) + k = g(x)$ has non-real roots.   (1)

                                                                                                    [17]

QUESTION 8

(a) $(x+3)+(x-3)+(12-x)+\dots$ is a convergent geometric series.

    (1)   Show that $x \neq -\dfrac{3}{2}$.   (3)
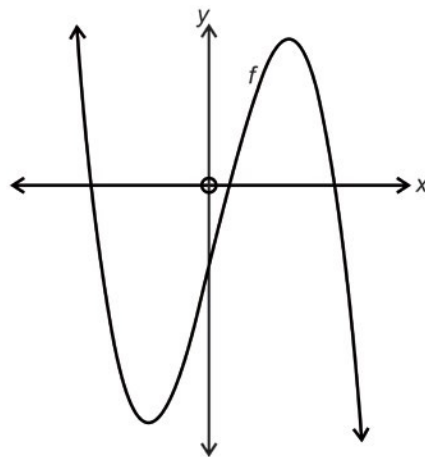
    (2)   Determine the value of x.   (4)

(b)   A geometric series is such that $S_4 = 7\dfrac{1}{2}$; $S_5 = 15\dfrac{1}{2}$ and $S_6 = 31\dfrac{1}{2}$.

    Determine, in terms of n, the sum to n terms of the series.   (7)

    [14]

QUESTION 9

The graph of a cubic function f is sketched below.

The equation of the curve is given as $f(x) = -x^3 + bx^2 + cx - 3$, where b and c are constants.

$f(1) = 4$ and $f''\left(\dfrac{1}{2}\right) = 1$

(a)   Determine the values of b and c.   (7)

(b)   Determine the values of x for which the graph is concave up.   (3)

    [10]

QUESTION 10

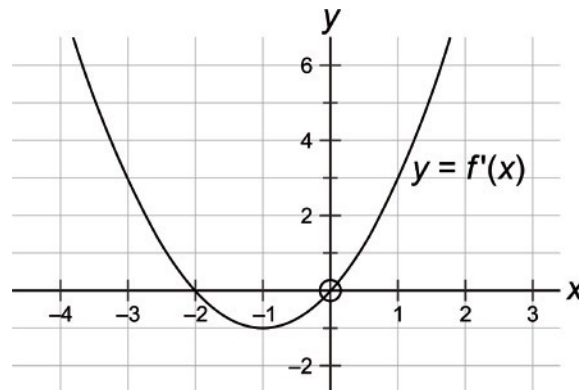**Round off your answer to the nearest whole number.**

A drink dispenser is able to fill up a 340 ml cup at a rate of *x* ml/s.
If the rate increases to $(x + 2)$ ml/s, the time taken to fill up the cup will be reduced
by 3 seconds.
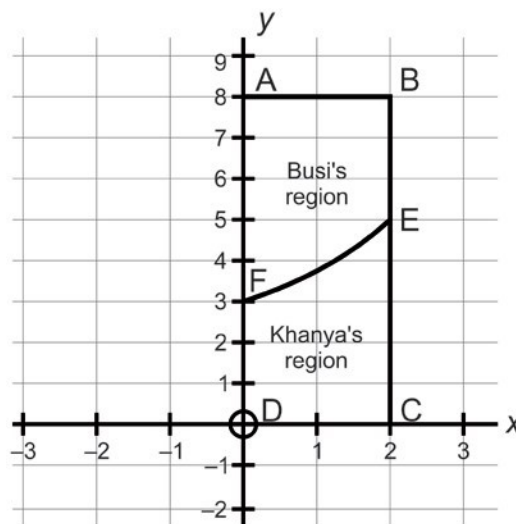
Determine the original time taken to fill the cup.

[6]

QUESTION 11

(a)   The sketch represents the graph of $y = f'(x)$ with x intercepts at $(-2;0)$ and $(0;0)$. The graph has a turning point at $(-1;-1)$.

(1)   Write down the value(s) of x for which the tangent to the graph of f is horizontal.   (2)

(2)   Draw a neat sketch of $y = f''(x)$.   (2)

(b)   Busi and Khanya have inherited land from their grandmother. This piece of land is represented on the section of the Cartesian plane as rectangle ABCD. A stream which can be modelled by the function $y = \dfrac{1}{15}x^3 + \dfrac{3}{4}x + 3$ for $x \in [0;2]$ flows through this piece of land from F to E as shown in the diagram. F $(0;3)$ and E $(2;5\dfrac{1}{30})$.

If Busi and Khanya build a straight road, which is a tangent to the curve at F, with the intention of dividing the piece of land into two regions, determine who will receive the larger region. Show all working.   (7)

                                                                                                    [11]

76 marks

Total: 150 marks