 MARKS


NATIONAL SENIOR CERTIFICATE EXAMINATION NOVEMBER 2022

## MATHEMATICS: PAPER I

## EXAMINATION NUMBER

$\square$
Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in DEGREE mode.
7. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers. Answers only will NOT necessarily be awarded full marks.
8. Round off to ONE DECIMAL PLACE unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. ONE blank page (page 24) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 12 | 13 | 17 | 9 | 13 | 9 | 8 | 14 | 22 | 14 | 13 | 6 | 150 |

## SECTION A

## QUESTION 1

(a) Solve for $x$, leaving answers correct to one decimal place if necessary.
(1) $(3 x-1)(4-x)=0$
(2) $2^{3 x}=7$
(3) $x(x-1)<20$
(b) Determine the value(s) of $p$ for which the roots of $x(x-6)=2 p$ are real and equal.

## QUESTION 2

(a) Solve for $x$ if $(x+3)^{\frac{1}{3}}=-2$.
(b) Solve for $x$ if $\log _{3}(x+5)-\log _{3} x=1$.
(c) (1) If $w=\sqrt{7-x}$; then for what value(s) of $x$ is $w$ non-real?
(2) Solve for $x$ if $w+2=x+1$.
(5)
[13]

## QUESTION 3

(a) Using first principles, find $g^{\prime}(x)$ if $g(x)=-3 x^{2}$.
(b) Find $f^{\prime}(x)$ if $f(x)=\frac{5}{3 x}+\sqrt[3]{x^{5}}$.
(c) In the diagram below:

- $f(x)=x^{2}-2 x-3$
- A is the $y$-intercept and B an x -intercept of $f$.
(1) Calculate the coordinates of $A$ and $B$.

(2) Determine the coordinates of the point on $f$ where the tangent to $f$ is parallel to line $A B$.


## QUESTION 4

(a) The $n$th term of an arithmetic sequence is given by $T_{n}=5 n-2$.
(1) Determine the number of terms in the sequence if the last term is 198.
(2) Determine the sum of all the terms of the sequence.
(b) The sum of the first $n$ terms of a geometric series is given by $S_{n}=8-2^{3-n}$. Determine the $9^{\text {th }}$ term of the series.

## QUESTION 5

In the diagram below:

- $g(x)=x^{3}-3 x$
- A and B are the turning points of $g$.
- C, O and D are the three $x$ intercepts of $g$.

(a) Determine the coordinates of A and B.
(b) On the Cartesian plane above, sketch $g^{\prime}(x)$.
(c) Determine the equation of the tangent to the original function $g(x)=x^{3}-3 x$ at $x=3$.


## QUESTION 6

You start a business and buy a delivery truck for R450 000.
(a) If the inflation rate is $6 \%$ per annum, how much will a new delivery truck cost in five years' time?
(b) If you calculate depreciation at 20\% per annum using the reducing-balance method, what will the value of your delivery truck be in five years' time?
(c) You set up a sinking fund for the next 5 years so that you can trade in your delivery truck and buy a new one by financing the difference with the sinking fund.

The bank offers an interest rate of $9 \%$ per annum compounded monthly. How much should you put away at the end of each month so that you have enough to finance the difference between the new vehicle and trade-in value of your delivery truck at the end of 5 years? The last payment into the sinking fund is made at the end of the 5 years.

## SECTION B

## QUESTION 7

A person is able to pay R14 500 per month to pay off a loan. The bank offers the person an interest rate of $12 \%$ per annum compounded monthly.
(a) What is the maximum loan the person can get from the bank if it needs to be paid back over a period of 20 years? (Round down to the nearest hundred.)
(b) If the loan above is taken up by the person, what will the outstanding balance be at the end of the 8th year (immediately after the instalment has been made), if they made payments of R14 500 per month?

## QUESTION 8

(a) Calculate the value of $x$ if:

$$
\sum_{n=3}^{x}(3 n+5)=711
$$

(b) In the diagram below, a ball bounces indefinitely along the path modelled below. The ball starts at $A$ where it is catapulted into the air before bouncing for the first time at $B$. The horizontal distance $B C=\frac{3}{4}$ of $A B$ and $C D=\frac{3}{4}$ of $B C$. This pattern continues indefinitely.


- Each bounce is modelled by the equation $y=-\frac{1}{2}(x-p)^{2}+q$.
- If the ball continued to bounce, it would eventually converge with the horizontal 16 metres from point $A$.
(1) Determine the length of $A B$.
(2) What is the maximum height reached by the ball between points $B$ and $C$ ?


## QUESTION 9

(a) In the diagram below:

- $w(x)=-x+6$ and $t(x)=x-4$ are the axes of symmetry for the given graph $h$.
- $E(9 ; 2)$ lies on $h$.
- $A B C D$ is a rectangle with $A D$ on the $x$-axis.

(1) If $h(x)=\frac{a}{x-p}+q$, find the values of $a, p$ and $q$.
(2) Determine the area of rectangle ABCD.
(b) (1) On the set of axes below sketch $g(x)=\log _{2} x$ if $x \leq 4$.

(2) On the set of axes above, sketch the graph of $g^{-1}(x)$ the inverse of $g$.
(3) Write down the domain of $g^{-1}(x)$.
(4) For what values of $x$ is $\frac{g^{-1}(x) \times g^{\prime}(x)}{g(x)} \leq 0$ ?


## QUESTION 10

You are given the following numbers $1 ; 3 ; 6 ; 8$; and 9 to use in creating five-digit numbers.
(a) (1) How many unique five-digit numbers can be created if repetition is not allowed?
(2) What is the probability that a unique five-digit number starts with an 8 and ends with a 3 , if repetition is not allowed?
(3) How many unique even numbers greater than 65000 can be created, if repetition is not allowed?
(b) In a factory that produces kettles, it is known that the probability of producing a defective kettle is $\frac{1}{65}$.
(1) How many kettles would you expect to be defective if 19500 kettles were produced by the factory?
(2) If you produced 150 kettles, what is the probability that none of these kettles would be defective? (Round off to four decimal places.)
(c) It is given that the $P(A)=x ; P(B)=0,6+x$ and the $P(A \cap B)=0,36-x$. If the events of $A$ and $B$ are independent, find the value of $x$.

## QUESTION 11

In the diagram below, C and D represent two drones on the ground.

- The drone at $C$ has a flight path modelled by $f(x)=-x^{2}+2 x+q$.
- The drone at D has a flight path modelled by $g(x)=m x+5$.
- $C D$ is 7 metres and $A B$ is 2 metres.

(a) If point E is directly above C , calculate the length of EC . Show all your workings.
(b) A friend of yours says: 'The vertical distance between flight path DB and the parabolic flight path CA will never be less than $40 \mathrm{~cm} . '$

Is your friend's statement correct? (Show all working.)

## QUESTION 12

Below is the sketch of $f^{\prime}(x)$ :

- A and B lie on $f^{\prime}(x)$ with C being the turning point.
- C lies on the $x$-axis.


If $f(0)=-14$ and $f(2)=2$, calculate the $x$ intercept for $f(x)$.

## ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

