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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2021

MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours — 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs et cetera that you have used in determining your answers.

   Answers only will NOT necessarily be awarded full marks.
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless stated otherwise.
8. There are two blank pages at the end of the paper that you may use if you run out of space when answering a question. If you use this space, clearly indicate this at the question and write the question number of your answer in the extra space.
9. It is in your own interest to write legibly and to present your work neatly.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
19	6	17	14	15	23	13	16	21	6	/150

SECTION A

QUESTION 1

(a) Given: $f(x) = -3x^2 + 1$

(1) Determine the average gradient of $f(x)$ between $x = 1$ and $x = 2$.

(3)

(2) Determine $f'(x)$ from first principles.

(5)

(b) Given: $g(x) = 2x^2 + \dfrac{2}{x}$

(1) Determine $g'(x)$.

(2)

(2) Determine $g\left(\dfrac{1}{x}\right)$.

(2)

(3) Hence, determine $g'(x) + g\left(\dfrac{1}{x}\right)$.

(2)

(c) A function f is given by $f(x) = 2x^3 - 5x^2 + x - 1$.

(1) Determine the gradient of the tangent to the function at the point where $x = 2$.

(3)

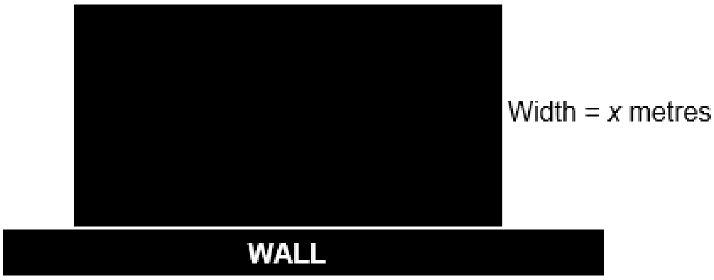
(2) Hence or otherwise determine the equation of the tangent to the function at the point where $x = 2$.

(2)
[19]

QUESTION 2

The diagram below represents a plan of the top view of a vegetable garden. The rectangular garden will be built against a wall and the rest of the vegetable garden's boundary will be built using a total of 30 metres of fencing.

The width of the garden is given as x metres.



(a) Which of the following equations represents the area, A, of the vegetable garden?

(1) $A = 15x - x^2$
(2) $A = 15x - 2x^2$
(3) $A = 30x - x^2$
(4) $A = 30x - 2x^2$

(2)

(b) Determine the dimensions of the vegetable garden that will ensure a maximum area.

Answers only will not be awarded full marks.

(4)
[6]

QUESTION 3

(a) The quadratic equation $9x^2 + bx - 5 = 0$ has roots $-\frac{1}{3}$ and $\frac{5}{3}$.

Determine the value of b.

(3)

(b) Solve for x in each case showing all working:

(1) $\sqrt{8x+12} + 1 = x$

(5)

(2) $x^2 < x$

(3)

(c) Solve for x in terms of p:

$$16^{\left(\frac{p}{4}\right)} = \frac{2^{(4x+1)}}{8^x}$$

(3)

(d) Given: $x^2 + px = 1$, where p is a constant.

Show that the roots of this equation can never be equal for any real number p.

(3)
[17]

QUESTION 4

Simon wants to buy a car that costs **R345 000**.

(a) He opens a savings account and six months after opening the account, he makes his first deposit of R12 895 and continues depositing R12 895 at the end of every six-month period.

Interest is paid at 13% per annum compounded half-yearly.

(1) How much money will be in Simon's account three years after opening the account?

(3)

(2) Ignoring the effect of inflation on the price of the car, determine how long it will take Simon to save the money needed to buy the car.

(5)

(3) If the effect of inflation is considered, determine the cost of the car eight years after opening the bank account.
Inflation for this period is calculated at 3,5% per annum.

(3)

(b) Instead of the savings plan, he considers a second plan which is getting a loan for R345 000 under the following agreement:

- Interest is charged at 13% per annum compounded half-yearly.
- The loan must be settled in eight years.

Determine his minimum monthly repayment.

(3)
[14]

QUESTION 5

(a) A hockey team will play two matches on a given day. The probability of the team winning, losing or drawing is 0,65; 0,15 and 0,2 respectively in each match.

(1) Draw a probability tree diagram to represent the possible results in the two matches.

(3)

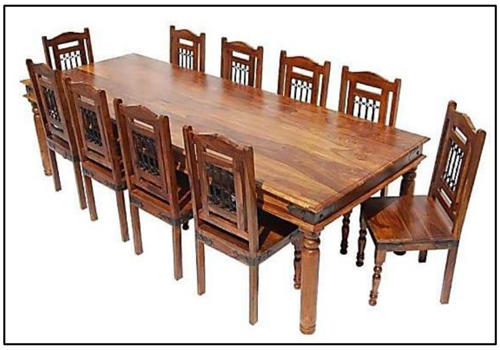
(2) What is the probability of the team having a win and a draw in any order?

(3)

(3) The coach claims that the probability of the team winning both matches is 1,3. Give a reason why this is incorrect.

(1)

(b) A venue is set up at a school event with rectangular tables that seat ten students.



[Source: <www.sierralivingconcepts.com>]

(1) In how many ways can the ten students be seated at a given table?

(2)

(2) Assuming that an extra ten students attend the event and the only table that is available can seat four of them.

In how many ways can the extra ten students be seated considering that only four chairs are available?

(3)

(c) A bench has five boys and four girls seated in a row with the girls occupying even places. How many arrangements are possible?

(3)
[15]

71 marks

SECTION B

QUESTION 6

(a) Given: A set of natural numbers up to and including 900.

If the multiples of 5 are removed from the set, determine the sum of the remaining numbers.

(7)

(b) If the sum of the first n terms of the following geometric series is to be greater than 300, determine the smallest value of n.

$$49+42+36+\frac{216}{7}+...$$

(5)

(c) For which value(s) of t is the following infinite series convergent?

$$2(t-5)+2(t-5)^2+2(t-5)^3+\ ...$$

(4)

(d) A wholesaler sells beach towels in packs of 5 for R200 per pack. If the number of towels purchased by a certain shopping outlet is $(100 + 5n)$, the price per towel would be reduced to $\text{R}(40 - n)$, where $n \in \mathbb{N}$.

(1) Show that the pattern that emerges for the total cost (T_n) for the number of towels purchased has a constant second difference of -10.

(3)

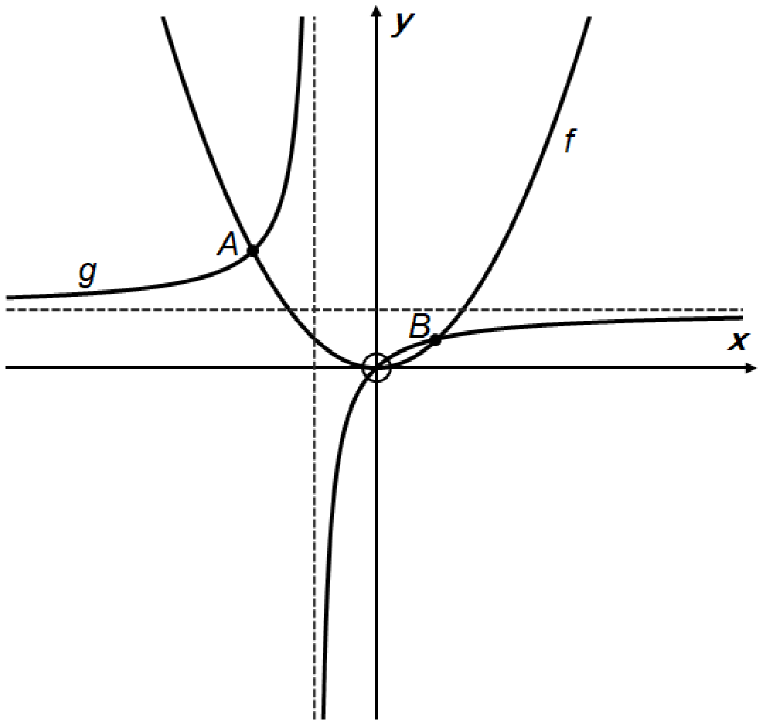
(2) Determine the n^{th} term in the form $T_n = an^2 + bn + c$ and show why this business model cannot be sustained indefinitely.

(4)
[23]

QUESTION 7

In the diagram below, the graphs of $f(x)=\frac{1}{2}x^2$ and $g(x)=-\frac{1}{x+1}+1$ are given.

A and B are <u>two</u> of the points of intersection of the graphs of f and *g.*



(a) Determine the coordinates of A and B.

(6)

(b) Determine x if $g(x) - f(x) > 0$.

(3)

(c) Determine g^{-1}, the inverse of g(x) in the form $y = \dfrac{a}{x+p} + q$.

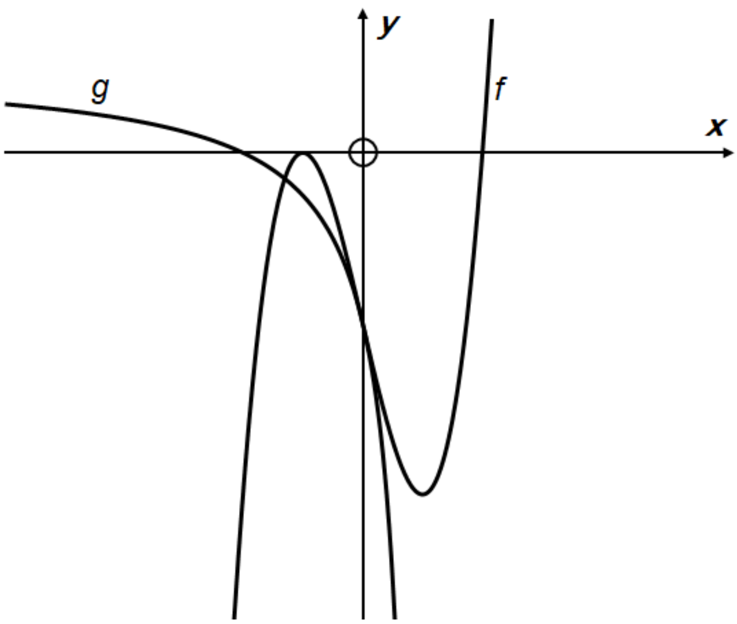
(4)
[13]

QUESTION 8

(a) A function is defined by $g(x) = 3x^3 - 4k^2x + 5$ where k is a positive number. Determine the value(s) of x in terms of k for which the function is decreasing.

(5)

(b) In the diagram below, the graphs of $g(x) = \dfrac{a}{x-1} + q$ for $x < 1$ and $f(x) = x^3 - 3x + t$ are given.
The graph of g has an x-intercept of $(-2;0)$.
Graphs f and g have a common y-intercept of $(0;-2)$.



(1) Determine the values of a and q.

(6)

(2) Determine the value(s) of x for which the graph of f is concave down.

(3)

(3) For what value(s) of k would $f(x) = k$ have three unique real roots?

(2)
[16]

QUESTION 9

(a) A function $f(x) = -px^2 + 2px$ has $p > 1$.

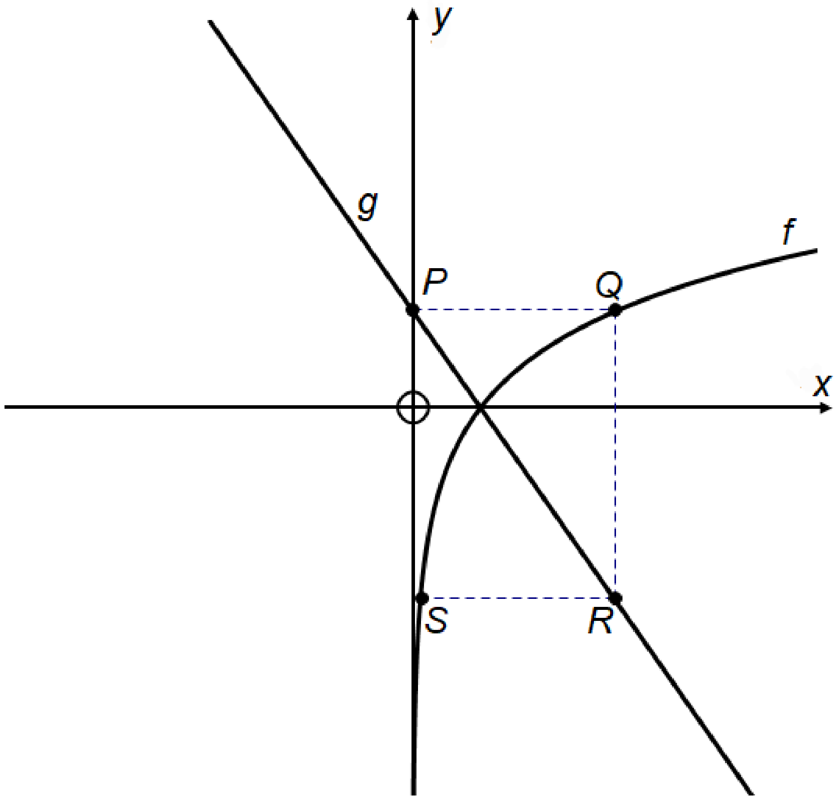
(1) On one set of axes, sketch the graphs of $f(x)$ and $f'(x)$. Show all intercepts and turning points, expressing these in terms of p if necessary.

(8)

(2) Show that the points of intersection of $f(x)$ and $f'(x)$ will always be independent of the value of p.

(3)

(b) In the diagram below, the graphs of $f(x) = \log_3 x$ and $g(x) = -x + 1$ are given.



(1) If $PQ \parallel SR \parallel x$-axis and $QR \parallel y$-axis, determine the coordinates of S.

(6)

(2) The graph with equation $k(x) = a.(3^x) + q$ has an asymptote passing through S and parallel to the x-axis.

Determine the values of a and q.

(4)
[21]

QUESTION 10

John and Simon are competing in a cycle race of 150 km. Simon cycles on average 4 km/h faster than John and finishes half an hour earlier than John.

Determine John's average speed.

Answers only will not be awarded full marks.

[6]

79 marks

Total: 150 marks

ADDITIONAL SPACE TO ANSWER QUESTIONS. REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE ALL ANSWERS ARE MARKED.