

NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2022

## MATHEMATICS: PAPER I

## EXAMINATION NUMBER



Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator unless stated otherwise.
6. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.
7. Diagrams are not necessarily drawn to scale.
8. If necessary, round off answers to ONE decimal place, unless stated otherwise.
9. It is in your own interest to write legibly and to present your work neatly.
10. One blank page (page 24) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 16 | 9 | 9 | 11 | 14 | 14 | 8 | 13 | 19 | 6 | 13 | 7 | 11 | $/ 150$ |

## SECTION A

## QUESTION 1

(a) Solve for $x$ if $\sqrt{2 x-7}=2$
(b) Solve for $x$ if $(3 x-1)^{2}=(x+1)^{2}$
(c) Solve for $x$ if $49^{x}=7^{x-2}$
(d) Solve for $x$ if $\log _{2}\left(x^{2}-3 x+10\right)=3$
(e) Solve for $x$ in terms of $p$ for the equation $2 x^{2}+3 p x=-p^{2}$

## QUESTION 2

(a) Given: $\sum_{n=1}^{\infty} 49\left(7^{-n}\right)$
(1) Write down the first two terms of the series.
(2) Calculate $S_{\infty}$
(b) Consider the quadratic sequence: $2 ; 5 ; 6 ; 5 ; 2 ; \ldots$

Determine the $n^{\text {th }}$ term of the sequence in the form $T_{n}=a n^{2}+b n+c$

## QUESTION 3

If Khanya took a loan of R1 800000 to purchase a house at the start of 2008 and the bank charged her interest at $9,25 \%$ per annum compounded monthly over a 30-year contract period:
(a) Calculate her monthly repayments.
(Assume that she received the loan immediately and that the first payment was made at the end of the first month).
(b) Calculate her outstanding balance at the end of 12 years.

## QUESTION 4

(a) Given: $f(x)=\frac{3}{x+2}+1$
(1) Sketch the graph of $y=f(x)$.

(2) Write down the domain of $f$.
(b) In the sketch below, the graph of $f(x)=3^{x}$ is given. The graph $g$ is the reflection of the graph of $f$ in the line $y=x$.

(1) Determine the equation of $g$.
(2) Determine, with the aid of the sketches given above, the value(s) of $x$ for which:
(i) $\quad 3^{x}>0$
(ii) $\log _{3} x \leq 0$

## QUESTION 5

(a) If $f(x)=-2 x^{2}+3 x$, determine $f^{\prime}(x)$ from first principles.
(b) Determine $\frac{d y}{d x}$ if:
(1) $y=2 x^{3}-4 x+5$
(2) $y=\frac{1}{\sqrt[5]{x^{2}}}$
(c) Given: $f(x)=-\frac{1}{3} x^{3}+x^{2}+3 x+1$. For what value(s) of $x$ is the function increasing?

## QUESTION 6

(a) Consider a group of 40 students. 20 play cricket, 25 play basketball and 6 students play both cricket and basketball.

A student is selected at random from the group.
Let $C$ be the event that the student plays cricket and let $B$ be the event that the student plays basketball.
(1) Are events $C$ and $B$ mutually exclusive? Explain.
(2) Calculate $P(C$ or $B)$
(b) There are twenty treats in a box. Ten are chocolates and ten are marshmallows. Nelson randomly selects two treats without replacement.
(1) Draw a tree diagram showing all possible outcomes.
(2) Determine the probability that the two treats are different.
(c) Consider the letters of the word: S Q U A RE

If the letters are arranged in any order without repetition to form different words, determine the probability that the word formed will start with an S and end with an E .

## SECTION B

## QUESTION 7

(a) The market value of John's house has increased at a constant annual exponential rate of $4,5 \%$ since he bought it on 1 January 2008. On 1 January 2021, his house was worth R3 500000.

Determine what the value of his house was on 1 January 2008, when he bought it.
(b) Helen also bought a house on 1 January 2008. Her house cost R1 800 000. Its market value has also been steadily increasing, but at a higher constant annual exponential rate of $5 \%$.

If the houses continue to keep increasing in value at the respective rates, in which calendar year will the two houses be worth the same amount?

Show all working.

## QUESTION 8

(a) The numbers $7 ; x ; y$ are in a geometric sequence. If the sum of these three numbers is $(-2 x-7)$, determine the possible value(s) of $x$ and $y$.
(b) An arithmetic series has a common difference of 5 . The sum of the first $n$ terms is 160 and the sum of the first $2 n$ terms is 725 .

Determine the value of $n$.
(6)
[13]

## QUESTION 9

In the diagram below, the graphs of $f(x)=a x^{2}+b x+c$ and $g(x)=x+8$ are given.
Note: O represents the origin.


The equation of the axis of symmetry of the parabola is given as $x=-1$.
$f$ has an $x$-intercept of $(-4 ; 0)$ and a given point $(-2 ; 4)$.
(a) Determine the values of $a, b$ and $c$.
(b) (1) If it is given that $f(x)=-\frac{1}{2} x^{2}-x+4$, determine $p$ and $q$ by completing the square and writing $f$ in the form $f(x)=a(x+p)^{2}+q$
(2) Give the value(s) of $x$ for which $4<f(x)<\frac{9}{2}$
(c) Determine the value(s) of $x$ for which $f(x) \cdot g(x) \leq 0$
(d) Calculate the minimum length of a line segment MN, parallel to the $y$-axis, that can be drawn from $g$ to $f$.

## QUESTION 10

Determine the quadratic function of $x$ that has the following properties:

- The function is symmetrical to $y=a x^{2}-7 x+c$ about the $y$-axis
- The function is symmetrical about $x=1$
- The function has a value of -20 when $x=4$


## QUESTION 11

In the diagram below, the graph of $f(x)=-x^{3}+5 x^{2}-3 x+d$, where $d$ is a constant, is given. $(3 ; 0)$ is a stationary point.

Note: O represents the origin.

(a) Calculate the value of $d$.
(b) Determine the gradient of the tangent to the curve at the point of inflection.
(5)
(c) Determine the value(s) of $x$ for which the graph is concave up.
(d) The equation $f(x)-k+3=0$ has three real roots. What are the possible value(s) of $k$ ?

## QUESTION 12

In the diagram below, the graph of $f^{\prime}(x)$ is sketched for $x \in \mathbb{R}$.
Note: O represents the origin.

(a) Write down the gradient of $f$ at $x=0$.
(b) Write down the $x$-coordinate of the turning point of $f$.
(c) What is the gradient of $f$ at $x=1$ ?
(d) Calculate the point of inflection of $f$.

## QUESTION 13

(a) For what value(s) of $t$ does the graph of the quadratic function $y=x^{2}+2 t x-t$ never touch the line $g(x)=x-\frac{5}{2}$ ?
(b) A container has a capacity of $500 \mathrm{~cm}^{3}$. It is filled with water at a rate of $x \mathrm{~cm}^{3} / \mathrm{s}$. When the rate increases to $(x+3) \mathrm{cm}^{3} / \mathrm{s}$, the time taken to fill up the container will be reduced by 40 seconds.

Determine the original time taken to fill up the container.

ADDITIONAL SPACE (ALL questions)
REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

