NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $x = \frac{1}{3}$ or $x = 4$

(2) $3x = \log_2 7$

$x = 0{,}9$

(3) $x(x - 1) < 20$

$x^2 - x - 20 < 0$

$(x - 5)(x + 4) < 0$

$-4 < x < 5$

(b) $x^2 - 6x - 2p = 0$

$\Delta = (-6)^2 - 4(1)(-2p)$

$36 + 8p = 0$

$p = -4{,}5$

Alternate solution

$x^2 - 6x - 2p = 0$

$-2p = 9$ (Create a perfect square)

$p = -4{,}5$

QUESTION 2

(a) $(x + 3)^{\frac{1}{3}} = -2$

$x + 3 = -8$

$x = -11$

(b) $\log_3(x + 5) - \log_3 x = 1.$

$\frac{(x+5)}{x} = 3$

$3x = x + 5$

$2x = 5$

$x = 2{,}5$

(c) (1) $x > 7$

(2) $\sqrt{7 - x} + 2 = x + 1$

$\sqrt{7 - x} = x - 1$

$7 - x = x^2 - 2x + 1$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3 \quad x \neq -2$

QUESTION 3

(a) $g(x) = -3x^2$

$$g'(x) = \lim_{h \to 0} \frac{-3(x+h)^2 - (-3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-6x - 3h)}{h}$$

$g'(x) = -6x$ (Notation)

(b) $f(x) = \frac{5}{3x} + \sqrt[3]{x^5}$

$f(x) = \frac{5}{3}x^{-1} + x^{\frac{5}{3}}$

$f'(x) = -\frac{5}{3}x^{-2} + \frac{5}{3}x^{\frac{2}{3}}$

(c) (1) A(0; –3)

$x^2 - 2x - 3 = 0$

$x = -1$ or $x = 3$

B(3; 0)

(2) $m_{AB} = 1$

$f'(x) = 2x - 2$

$2x - 2 = 1$

$x = \frac{3}{2}$

$y = \left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right) - 3$

$y = -\frac{15}{4}$

QUESTION 4

(a) (1) $5n - 2 = 198$

$5n = 200$

$n = 40$

(2) $S_{40} = \frac{40}{2}[2(3) + (40 - 1)5]$

$S_{40} = 4020$

Alternative:

$S_n = \frac{n}{2}(a + l) = \frac{40}{2}(3 + 198)$

$S_{40} = 4020$

(b) $S_9 = 8 - 2^{3-9} = 7\frac{63}{64}$

$S_8 = 8 - 2^{3-8} = 7\frac{31}{32}$

$T_9 = 7\frac{63}{64} - 7\frac{31}{32} = \frac{1}{64}$

Alternate solution:

$S_1 = T_1 = 8 - 2^2 = 4$

$S_2 = T_1 + T_2 = 8 - 2 = 6 \quad \therefore T_2 = 2$

$T_9 = ar^8 = 4\left(\frac{1}{2}\right)^8 = \frac{1}{64}$

QUESTION 5

(a) $g(x) = x^3 - 3x$

$g'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$(x + 1)(x - 1) = 0$

$x = -1$ or $x = 1$

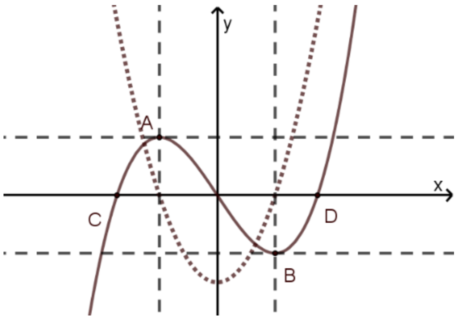
$A(-1; 2)$

$B(1; -2)$

(b) x intercepts

y intercept

Shape



(c) $g'(x) = 3x^2 - 3$

$g'(3) = 3(3)^2 - 3$

$g'(3) = 24$

$g(3) = (3)^3 - 3(3)$

$g(3) = 18$

$y = 24x + c$

$18 = 24(3) + c$

$c = -54$

$y = 24x - 54$

QUESTION 6

(a) $A = 450\ 000(1 + 0{,}06)^5$

$A = \text{R}602\ 201{,}\ 51$

(b) $A = 450\ 000(1 - 0{,}2)^5$

$A = \text{R}147\ 456$

(c) R602 201,51 – R147 456

= R454 745,51

$$454\ 745{,}51 = \frac{x[\left(1 + \frac{0{,}09}{12}\right)^{60} - 1]}{\frac{0{,}09}{12}}$$

$x = \text{R}6\ 029{,}18$

SECTION B

QUESTION 7

(a) $A = \dfrac{14\ 500[1 - \left(1 + \frac{0,12}{12}\right)^{-240}]}{\frac{0,12}{12}}$

Loan amount = R1 316 800

(b) $A = 1\ 316\ 800\left(1 + \dfrac{0,12}{12}\right)^{96}$

A = R3 422 722, 59

Future value of payments

$F_v = \dfrac{14\ 500[\left(1 + \frac{0,12}{12}\right)^{96} - 1]}{\frac{0,12}{12}}$

F_v = R2 318 945,74

Balance outstanding = R1 103 776, 85

QUESTION 8

(a) $14 + 17 + 20 + 23 + \ldots + (3x + 5) = 711$

$711 = \frac{n}{2}(2(14) + (n-1)(3))$ Or Alternate: $711 = \frac{x-2}{2}(14 + 3x + 5)$

$711 = \frac{n}{2}(3n + 25)$

$0 = 3n^2 + 25n - 1422$

$n = 18$ or $n \neq -\frac{79}{3}$

therefore

$x = 20$

OR

$8 + 11 + 14 + \ldots + (n \textit{ terms}) = 730$

$730 = \frac{n}{2}(2(8) + (n-1)(3))$

$0 = 3n^2 + 13n - 1460$

$n = 20$ or $n \neq -\frac{73}{3}$

therefore

$x = 20$

(b) (1) $16 = \frac{a}{1 - \frac{3}{4}}$

$AB = 4$

(2) BC = 3 metres

When $x = \frac{11}{2}$ the maximum height is achieved

$y = -\frac{1}{2}(\frac{11}{2} - 4)(\frac{11}{2} - 7)$

$y = \frac{9}{8}$ metres or 1,1 metres (Rounded off to one decimal place)

Maximum height between B and C is 1,125 metres.

QUESTION 9

(a) (1) $-x + 6 = x - 4$

$-2x = -10$

$x = 5$

$y = -(5) + 6$

$y = 1$

$h(x) = \frac{a}{x-5} + 1$

$2 = \frac{a}{9-5} + 1$

$a = 4$

$p = 5$ *and* $q = 1$

(2) $0 = \frac{4}{x-5} + 1$

$-x + 5 = 4$

$x = 1$

$A(1; 0)$

$x - 4 = \frac{4}{x-5} + 1$ (Hyperbola intercepts with axis of symmetry)

$x^2 - 9x + 20 = 4 + x - 5$

$x^2 - 10x + 21 = 0$

$(x - 7)(x - 3) = 0$

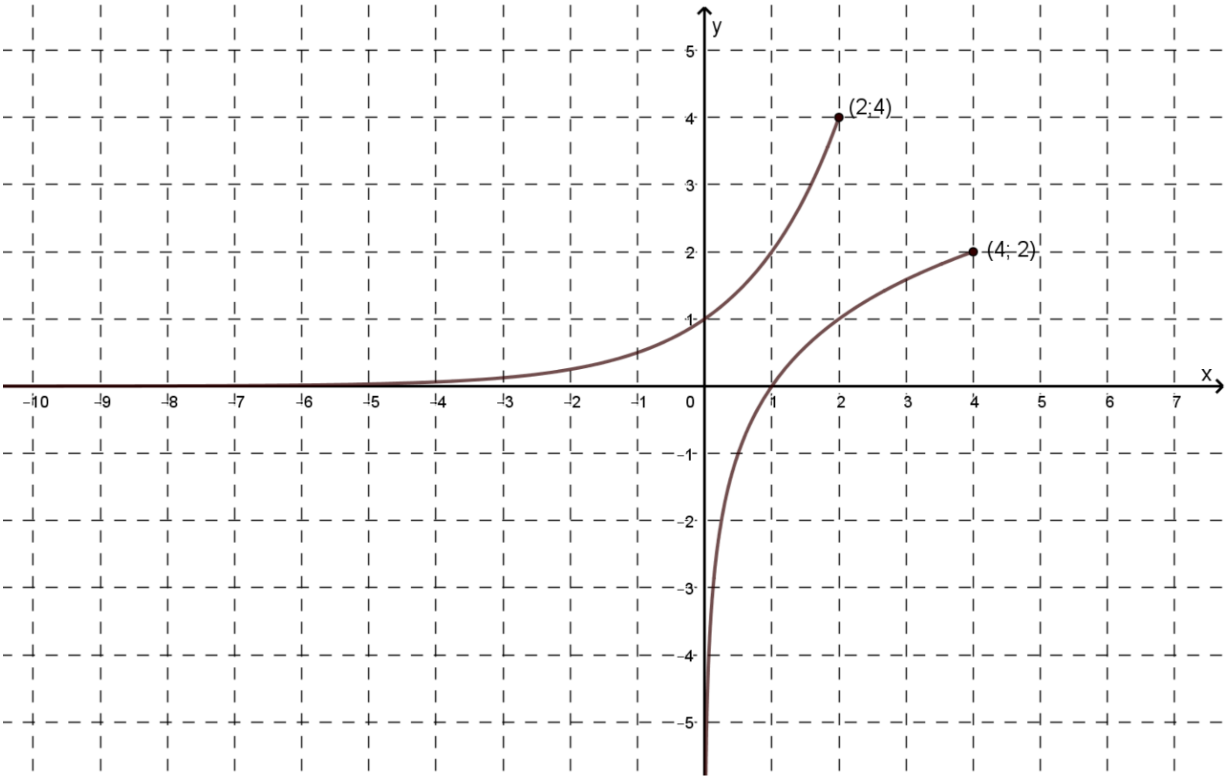
$x = 7$ or $x = 3$

$y = (7) - 4 = 3$

$C(7; 3)$

Area of rectangle = $6 \times 3 = 18$ units^2

(b) (1) x-intercept (4; 2) shape and asymptote



(2) x-intercept (2; 4) shape and asymptote

(3) $x \in (-\infty; 2]$

(4) $g'(x) > 0$

$$\frac{g^{-1}(x)}{g(x)} \leq 0$$

$0 < x < 1$ (Notation)

QUESTION 10

(a) (1) 5! or 120

(2) $\frac{3!}{120} = 0,05$ or $\frac{1}{20}$

(3) 6 9 _ _ 8 $2 \times 1 = 2$ for the split

8 _ _ _ 6 $3 \times 2 \times 1 = 6$

9 _ _ _ 6 $3 \times 2 \times 1 = 6$

9 _ _ _ 8 $3 \times 2 \times 1 = 6$

Total unique even numbers = 20.

Alternate: _ _ _ _ 6: $2 \times 3 \times 2 \times 1 = 12$

9 _ _ _ 8: $3 \times 2 \times 1 = 6$

6 _ _ _ 8: $1 \times 2 \times 1 = 2$

(b) (1) $19\ 500 \times \frac{1}{65} = 300$ kettles

(2) $\left(\frac{64}{65}\right)^{150} = 0,0977$

(c) $x(x + 0,6) = 0,36 - x$

$x^2 + 0,6x - 0,36 + x = 0$

$x^2 + 1,6x - 0.36 = 0$

$(x - 0,2)(x + 1,8) = 0$

$x > 0$ therefore $x = 0,2$

QUESTION 11

(a) $OB = 5$

Therefore $q = 3$ metres

$-x^2 + 2x + 3 = 0$
$0 = x^2 - 2x - 3$
$0 = (x - 3)(x + 1)$
$x = 3$ or $x = -1$

$C(3; 0)$

$D(10; 0)$

$y = -\frac{1}{2}x + 5$

$y = -\frac{1}{2}(3) + 5$

$y = \frac{7}{2}$

$E(3; \frac{7}{2})$

EC = $3\frac{1}{2}$ metres

(b) Vertical Distance = $-\frac{1}{2}x + 5 - (-x^2 + 2x + 3)$

Vertical Distance = $-\frac{1}{2}x + 5 + x^2 - 2x - 3$

Vertical Distance = $x^2 - \frac{5}{2}x + 2$

$\frac{d_{VD}}{d_X} = 2x - \frac{5}{2}$

$2x - \frac{5}{2} = 0$

$x = \frac{5}{4}$ or 1,25

Minimum vertical distance = $(1,25)^2 - \frac{5}{2}(1,25) + 2$

Minimum vertical distance = 0,4375 metres or 43,75 cm or 43,8 cm

Your friend's statement is correct.

QUESTION 12

$y = a(x-2)^3 + 2$

$-14 = a(0-2)^3 + 2$

$-16 = -8a$

$a = 2$

$0 = 2(x-2)^3 + 2$

$x = 1$

Alternate Solution:

$f'(x) = a(x-2)^2$

$54 = 9a \quad \therefore a = 6$

$f'(x) = 6x^2 - 24x + 24$

$f(x) = 2x^3 - 12x^2 + 24x - 14$

$f(1) = 0$

$\therefore x = 1$

Total: 150 marks