NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2021

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a student answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

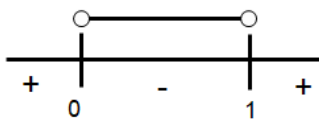
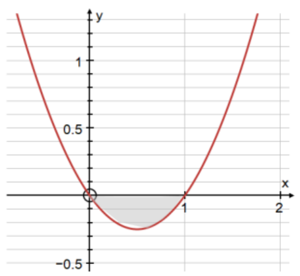
(a) (1)	$f(1) = -3(1)^2 + 1$ $f(1) = -2$ $f(2) = -3(2)^2 + 1$ $f(2) = -11$ $m = \dfrac{-11-(-2)}{2-1}$ $\therefore m = -9$	$f(1) = -2$ $f(2) = -11$ $\therefore m = -9$
(2)	$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x) = \lim\limits_{h\to 0}\dfrac{-3(x+h)^2+1-(-3x^2+1)}{h}$ $f'(x) = \lim\limits_{h\to 0}\dfrac{-3(x^2+2xh+h^2)+1+3x^2-1}{h}$ $f'(x) = \lim\limits_{h\to 0}\dfrac{-3x^2-6xh-3h^2+1+3x^2-1}{h}$ $f'(x) = \lim\limits_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $f'(x) = \lim\limits_{h\to 0}(-6x-3h)$ $= -6x$ **OR** $f(x+h) = -3(x+h)^2 + 1$ $f(x+h) = -3(x^2+2xh+h^2)+1$ $f(x+h) = -3x^2-6xh-3h^2+1$ $f(x+h)-f(x) = -3x^2-6xh-3h^2+1+3x^2-1$ $f(x+h)-f(x) = -6xh-3h^2$ $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x) = \lim\limits_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $f'(x) = \lim\limits_{h\to 0}(-6x-3h)$ $= -6x$	$-3(x+h)^2+1-(-3x^2+1)$ Simplification Factorisation $\lim\limits_{h\to 0}(-6x-3h)$ $= -6x$ $-3(x+h)^2+1-(-3x^2+1)$ Simplification Factorisation $\lim\limits_{h\to 0}(-6x-3h)$ $= -6x$

(b) (1)	$g(x) = 2x^2 + 2x^{-1}$ $g'(x) = 4x - 2x^{-2}$ $g'(x) = 4x - \frac{2}{x^2}$	$4x$ $-2x^{-2}$
(2)	$g\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right)^2 + \frac{2}{\frac{1}{x}}$ $g\left(\frac{1}{x}\right) = \frac{2}{x^2} + 2x$	correct subst $\frac{2}{x^2} + 2x$
(3)	$g'(x) + g\left(\frac{1}{x}\right)$ $= 4x - \frac{2}{x^2} + \frac{2}{x^2} + 2x$ $= 6x$	simplification $6x$
(c) (1)	$f'(x) = 6x^2 - 10x + 1$ $f'(x) = 6(2)^2 - 10(2) + 1$ $f'(2) = 5$	$6x^2 - 10x + 1$ subst $f'(2) = 5$
(2)	$f(2) = -3$ Eq. of tangent: $y = 5x + c$ Subst: $(2;-3)$ $c = -13$ $\therefore y = 5x - 13$	$f(2) = -3$ $c = -13$

QUESTION 2

(a)	Answer: Option 4	answer
(b)	$\frac{dA}{dx} = 30 - 4x$ $30 - 4x = 0$ $x = 7\frac{1}{2}$ Therefore the dimensions are: $7\frac{1}{2}$ by 15	$30 - 4x$ $30 - 4x = 0$ answer

QUESTION 3

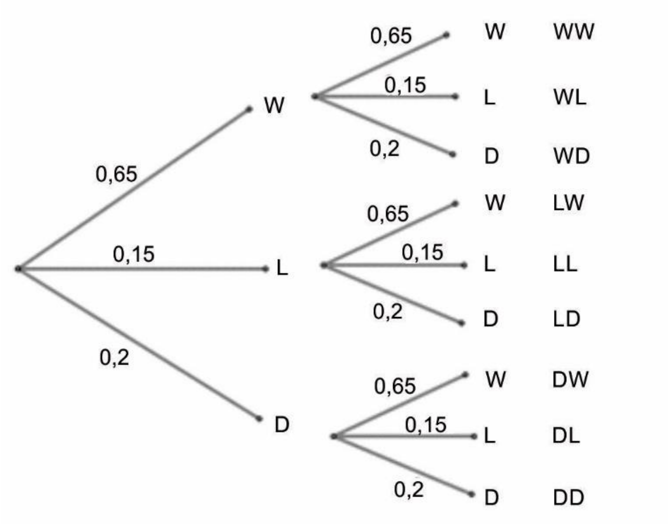
(a)	$x=-\frac{1}{3}$ or $x=\frac{5}{3}$ $\left(x+\frac{1}{3}\right)\left(x-\frac{5}{3}\right)=0$ $x^2-\frac{4}{3}x-\frac{5}{9}=0$ LCD: 9 $9x^2-12x-5=0$ $\therefore b=-12$ OR $(3x+1)(3x-5)=0$ $9x^2-12x-5=0$ $\therefore b=-12$	$\left(x+\frac{1}{3}\right)\left(x-\frac{5}{3}\right)=0$ $9x^2-12x-5=0$ $\therefore b=-12$ $(3x+1)(3x-5)=0$ $9x^2-12x-5=0$ $\therefore b=-12$
(b) (1)	$\sqrt{8x+12}=x-1$ $8x+12=x^2-2x+1$ $x^2-10x-11=0$ $(x-11)(x+1)=0$ $x=11$ or $x=-1$ Check: $x=-1$ is not valid	Isolate surd x^2-2x+1 $x^2-10x-11$ factors answer with selection
(2)	$x^2-x<0$ $x(x-1)<0$ Crit. values: 0 ; 1   Solution: $0<x<1$	Factors/critical values Number line /graph $0<x<1$

(c)	$\left(2^4\right)^{\frac{p}{4}} = \dfrac{2^{4x+1}}{2^{3x}}$ $2^p = 2^{x+1}$ $x = p - 1$	2^p 2^{x+1} answer
(d)	$x^2 + px - 1 = 0$ $x = \dfrac{-p \pm \sqrt{p^2 + 4}}{2}$ For the roots to be equal, $p^2 + 4 = 0$ $p^2 = -4$ Therefore no real solution **OR** $\Delta = 0$ for equal roots $p^2 + 4 = 0$ $p^2 = -4$ Therefore no real solution	$\dfrac{-p \pm \sqrt{p^2 + 4}}{2}$ $p^2 = -4$ Therefore no real sol. $\Delta = 0$ for equal roots $p^2 = -4$ Therefore no real sol.

QUESTION 4

(a) (1)	$F = 12\,895\left[\dfrac{\left(1+\dfrac{13}{200}\right)^{3\times 2}-1}{\dfrac{13}{200}}\right]$ $F = \text{R}91\,086{,}76791$	Subst. into correct formula Accurate substitution answer
(2)	$345\,000 = 12\,895\left[\dfrac{\left(1+\dfrac{13}{200}\right)^{2n}-1}{\dfrac{13}{200}}\right]$ $2{,}739\,046\,142 = (1{,}065)^{2n}$ $\log_{1{,}065} 2{,}739\,046\,142 = 2n$ $n \approx 8$ years	Subst. into correct formula Accurate substitution simplification conversion to log form answer
(3)	$A = 345\,000\left(1+\dfrac{3{,}5}{100}\right)^{8}$ $A = \text{R}454\ \ 299{,}1178$	Subst. into correct formula Accurate substitution Answer
(b)	$P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $345\,000 = \dfrac{x\left[1-\left(1+\dfrac{13}{200}\right)^{-8\times 2}\right]}{\dfrac{13}{200}}$ $x = \text{R }35\,320{,}26301$	Subst. into correct formula Accurate substitution Answer

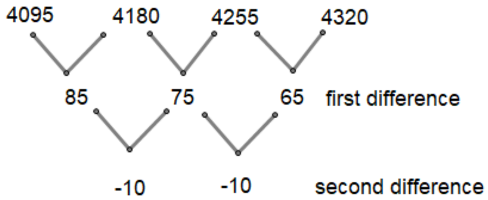
QUESTION 5

(a) (1)		Tree diagram representing 2 matches correct probabilities indicated correct labels
(2)	P(win & draw in any order) = P(WD) + P(DW) $=(0,65\times 0,2)+(0,2\times 0,65)$ $=\frac{13}{50}$ $=0,26$	$(0,65\times 0,2)$ $(0,2\times 0,65)$ $=0,26$
(3)	He is not correct because the probability of an event occurring cannot be greater than 1	justification
(b) (1)	$10!$ $=3\,628\,800$	$10!$ $=3\,628\,800$
(2)	$10\times 9\times 8\times 7$ $=5\,040$ **OR** $\frac{10!}{(10-4)!}$ $=5\,040$	$10\times 9\times 8\times 7$ $=5\,040$ $10!$ $(10-4)!$ $=5\,040$
(c)	$5!\ \times\ 4!$ $=2\,880$	$5!$ $4!$ $=2\,880$

SECTION B

QUESTION 6

(a)	$S_n = \frac{n}{2}(a+l)$ $S_{900} = \frac{900}{2}(1+900)$ $S_{900} = 405\,450$ No. of terms that are multiples of 5 : 180 $S_{180} = \frac{180}{2}(5+900)$ $S_{180} = 81\,450$ Sum of remaining numbers $= 405\,450 - 81\,450$ $= 324\,000$	Correct formula $300 = \frac{n}{2}(3+47)$ $S_{900} = 405\,450$ 180 $(5+900)$ $S_{180} = 81\,450$ $324\,000$
(b)	$S_n = \frac{a(r^n-1)}{r-1}$ $a = 49$ and $d = \frac{6}{7}$ $300 = \frac{49\left(\left(\frac{6}{7}\right)^n - 1\right)}{\frac{6}{7}-1}$ $\left(\frac{6}{7}\right)^n = \frac{43}{343}$ $\log_{\left(\frac{6}{7}\right)}\left(\frac{43}{343}\right) = n$ $n \approx 13,47$ $\therefore n = 14$	$d = \frac{6}{7}$ Substitution into correct formula simplification $\log_{\left(\frac{6}{7}\right)}\left(\frac{138}{343}\right) = n$ Answer
(c)	$r = t-5$ $-1 < t-5 < 1$ $4 < t < 6$	$r = t-5$ $-1 < \overset{r}{x} < 1$ $t > 4$ $t < 6$

(d) (1)	$n=1$: $T_n=[100+5(1)]\times(40-1)$ $=4\,095$ $n=2$: $T_2=4\,180$ $T_3=4\,255$ $T_4=4\,320$  	4 095, 4 180, 4 255, 4 320 first difference terms second difference
(2)	$T_n=(100+5n)(40-n)$ $T_n=-5n^2+100n+4\,000$ When $n=40$ total cost will be zero. **OR** $2a=-10$ $3a+b=85$ $a+b+c=4\,095$ $a=-5$ $b=100$ $c=4\,000$ $T_n=-5n^2+100n+4\,000$ When $n=40$ total cost will be zero. **OR** $2a=-10$ $\therefore a=-5$ $T_n=-5n^2+bn+c$ $T_1: -5(1)^2+b(1)+c=4\,095$ $\therefore$ $b+c=4\,100$ $T_2: -5(2)^2+b(2)+c=4\,180$ $\therefore$ $2b+c=4\,200$ $T_2-T_1: b=100$ $\therefore c=4\,000$ $T_n=-5n^2+100n+4\,000$ When $n=40$ total cost will be zero.	$a=-5$ $b=100$ $c=4000$ $n=40$, cost = 0 $a=-5$ $b=100$ $c=4000$ $n=40$, cost = 0 $a=-5$ $b=100$ $c=4000$ $n=40$, cost = 0

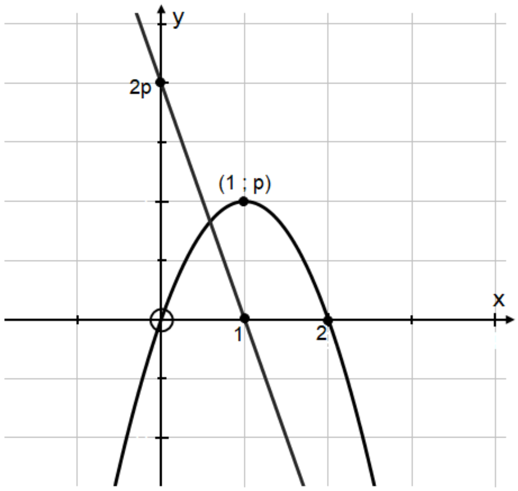
QUESTION 7

(a)	$\frac{1}{2}x^2 = -\frac{1}{x+1}+1$ LCD: $2(x+1)$ Restr.: $x \neq -1$ $x^2(x+1) = -1(2)+2(x+1)$ $x^3+x^2 = -2+2x+2$ $x^3+x^2-2x=0$ $x(x^2+x-2)=0$ $x(x+2)(x-1)=0$ $x=0$ or $x=-2$ or $x=1$ $\therefore A(-2;2)$ and $B\left(1;\frac{1}{2}\right)$	Equating for points of intersection simplification x^3+x^2-2x factorisation $x(x+2)(x-1)$ $x=0$ or $x=-2$ or $x=1$ $A(-2;2)$ $B\left(1;\frac{1}{2}\right)$
(b)	$g(x) > f(x)$ for: $-2 < x < -1$ or $0 < x < 1$	$-2 < x < -1$ $0 < x < 1$ $x < 1$
(c)	$g: y = -\frac{1}{x+1}+1$ $g^{-1}: x = -\frac{1}{y+1}+1$ $\frac{1}{y+1} = -x+1$ $(y+1)(-x+1)=1$ $y(-x+1) = 1-1+x$ $y = \frac{-(-x+1)+1}{(-x+1)}$ $y = -\frac{1}{x-1}-1$	Changing x and y $(y+1)(-x+1)=1$ $y = \frac{-(-x+1)+1}{(-x+1)}$ $y = -\frac{1}{x-1}-1$

QUESTION 8

(a)	Function decreases when: $g'(x) < 0$ $g'(x) = 9x^2 - 4k^2$ $9x^2 - 4k^2 < 0$ $\therefore (3x - 2k)(3x + 2k) < 0$ Crit. Values: $\pm \frac{2k}{3}$ $-\frac{2k}{3} < x < \frac{2k}{3}$	$g'(x) < 0$ $9x^2 - 4k^2$ $\therefore (3x - 2k)(3x + 2k) < 0$ Crit. Values: $\pm \frac{2k}{3}$ answer
(b) (1)	$g(x) = \frac{a}{x-1} + q$ subst. $(-2;0)$ $0 = \frac{a}{-2-1} + q$ $q = \frac{a}{3}$ ... eq.1 $g(x) = \frac{a}{x-1} + q$ subst. $(0;-2)$ $-2 = \frac{a}{0-1} + q$ $-2 = -a + q$ ... eq.2 Subst. eq.1 in eq.2 $-2 = -a + \frac{a}{3}$ $a = 3$ $\therefore q = 1$	subst. $(-2;0)$ $q = \frac{a}{3}$ subst. $(0;-2)$ $-2 = -a + q$ $a = 3$ $q = 1$
(2)	$f(x) = x^3 - 3x + t$ $f'(x) = 3x^2 - 3$ $f''(x) = 6x$ For concave down: $f''(x) < 0$ $6x < 0$ $x < 0$	$f'(x) = 3x^2 - 3$ $f''(x) < 0$ $x < 0$
(3)	$f(x) = x^3 - 3x - 2$ $f(x) - k = 0$ has 3 real roots for: $-4 < k < 0$	$f(x) = x^3 - 3x - 2$ $-4 < k < 0$

QUESTION 9

(a) (1)		max. value parabola T.P (1 ; p) X-int: (0;0) X-int: (2;0) Y-int: (0;0) straight line with negative gradient X-int: (1;0) Y-int: (0;2p)
(2)	$f(x) = f'(x)$ $-px^2 + 2px = -2px + 2p$ $-px^2 + 4px - 2p = 0$ $-p(x^2 - 4x + 2) = 0$ $\therefore$ For point of intersect.: $(x^2 - 4x + 2) = 0$ $\therefore$ Independent of p OR $-px^2 + 2px = -2px + 2p$ $-px^2 + 4px - 2p = 0$ $-p(x^2 - 4x + 2) = 0$ $\therefore$ For point of intersection at: $x = 2 \pm \sqrt{2}$ $\therefore$ Independent of p	$-px^2 + 2px = -2px + 2p$ $-p(x^2 - 4x + 2) = 0$ $(x^2 - 4x + 2) = 0$ $-px^2 + 2px = -2px + 2p$ $-p(x^2 - 4x + 2) = 0$ $x = 2 \pm \sqrt{2}$
(b) (1)	$g(x) = -x + 1 \;\therefore\; y$-intercept: $(0;1)$ $\therefore P(0;1)$ For Q : subst.: $(x;1)$ in $f(x) = \log_3 x$ $1 = \log_3 x$ $x = 3$ $\therefore Q(3;1)$ For R : subst.: $(3;y)$ in $g(x) = -x + 1$ $y = -3 + 1$ $y = -2$ $\therefore R(3;-2)$ For S : subst.: $(x;-2)$ in $f(x) = \log_3 x$	$\therefore P(0;1)$ $1 = \log_3 x$ $\therefore Q(3;1)$ $\therefore R(3;-2)$

	$-2=\log_3 x$ $3^{-2}=x$ $x=\frac{1}{9}$ $\therefore S\left(\frac{1}{9};-2\right)$	$3^{-2}=x$ $\therefore S\left(\frac{1}{9};-2\right)$
(2)	$k(x)=a.3^x-2$ Subst.: x-int. (0;–1) $-1=a.b^0-2$ $a=1$ Alternate: If $a<0$ $q=-2$ but y-int $(0;-3)$ $y=(3)^x-2$ $-3=a(3)^0-2$ $sub(0;-3)$ $-1=a$	$q=-2$ (0;–1) $-1=a.b^0-2$ $a=1$

QUESTION 10

Q 10	Distance = Speed $\times$ time Let John's speed = x Simon's speed = $x+4$ Simon: $150=(x+4)(t)$ ...eq.1 John: $150=x(t+0,5)$ ... eq.2 $\therefore$ eq.1 = eq.2 $tx+4t=tx+0,5x$ $4t=0,5x$ $x=8t$ …. Subst. in eq.1 $(8t+4)(t)=150$ $8t^2+4t-150=0$ $t\approx 4,09$ or $t\approx -4,59$ Not valid John's Speed: $150=x(4,09+0,5)$ $150=4,59x$ $x=32,7$ km/h OR $\frac{150}{x+4}+\frac{1}{2}=\frac{150}{x}$ LCD: $2x(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $\therefore x\approx 32,7$ km/h	correct formula $150=(x+4)(t)$ $150=x(t+0,5)$ $tx+4t=tx+0,5x$ $t\approx 4,09$ $x=32,7$ $\frac{150}{x+4}+\frac{1}{2}=\frac{150}{x}$ $2x(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $\therefore x\approx 32,7$

Total: 150 marks