NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2022

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a student answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)	$2x-7=4$ $x=\frac{11}{2}$	squaring $x=\frac{11}{2}$
(b)	$3x-1=\pm(x+1)$ $3x-1=x+1$ or $3x-1=-x-1$ $x=1$ or $x=0$ **OR** $9x^2-6x+1=x^2+2x+1$ $8x^2-8x=0$ $8x(x-1)=0$ $x=0$ or $x=1$	$3x-1=\pm(x+1)$ $x=1$ a $x=0$ Multiplying out $x=1$ $x=0$
(c)	$7^{2x}=7^{x-2}$ $\therefore 2x=x-2$ $\therefore x=-2$ **Alternate:** $7^{2x}-7^{x-2}=0$ $7^x.7^x-7^x.7^{-2}=0$ $7^x\left(7^x-\frac{1}{7^2}\right)=0$ $\therefore 7^x=0$ n/v or $7^x=7^{-2}$ $\therefore x=-2$	$7^{2x}=7^{x-2}$ $\therefore 2x=x-2$ $\therefore x=-2$ $7^{2x}=7^{x-2}$ $7^x\left(7^x-\frac{1}{7^2}\right)=0$ $\therefore x=-2$
(d)	$\log_2(x^2-3x+10)=3$ $x^2-3x+10=2^3$ $x^2-3x+2=0$ $(x-2)(x-1)=0$ $x=2$ or $x=1$	$x^2-3x+10=2^3$ $(x-2)(x-1)=0$ $x=2$ $x=1$
(e)	$2x^2+3px+p^2=0$ $(2x+p)(x+p)=0$ $x=-\frac{1}{2}p$ or $x=-p$	$(2x+p)(x+p)=0$ $x=-\frac{1}{2}p$ $x=-p$

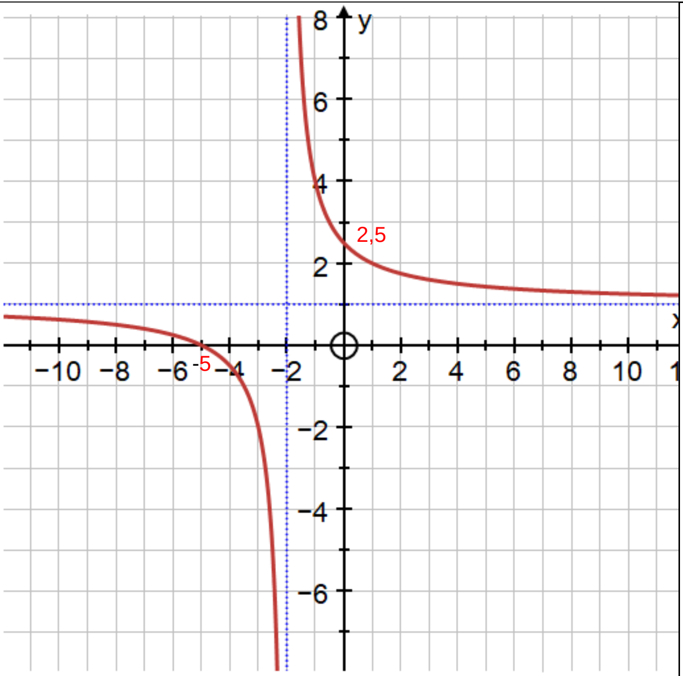
QUESTION 2

(a)(1)	$T_1 = 7$; $T_2 = 1$	$T_1 = 7$ $T_2 = 1$
(a)(2)	$S_\infty = \dfrac{7}{1-\frac{1}{7}}$ $S_\infty = \dfrac{49}{6}$	$S_\infty = \dfrac{7}{1-\frac{1}{7}}$ $S_\infty = \dfrac{49}{6}$
(b)	2 5 6 5 2 Sequence 3 1 –1 –3 First difference –2 –2 –2 Constant second differ $2a = -2 \quad \therefore a = -1$ $3a + b = 3 \quad \therefore \quad b = 6$ $a + b + c = 2 \quad \therefore \quad c = -3$ $T_n = -n^2 + 6n - 3$	First difference Constant second differ. $a = -1$ $b = 6$ $c = -3$

QUESTION 3

(a)	$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$ $1\,800\,000 = x\left[\frac{1-\left(1+\frac{9,25}{1200}\right)^{-30\times12}}{\frac{9,25}{1200}}\right]$ $x = R14\,808,16$	Use of correct formula $\left(1+\frac{9,25}{1200}\right)$ $n = 360$ in Pv formula $x = R14\,808,16$
(b)	OB = A – F $A = 1800\,000\left(1+\frac{9,25}{1200}\right)^{144}$ $A = 5\,438\,647,444$ $F = 14\,808,15766\left[\frac{\left(1+\frac{9,25}{1200}\right)^{144}-1}{\frac{9,25}{1200}}\right]$ $F = 3\,883\,363,245$ OB $= 1555\,284,20$ **OR** OB $= 14\,808,16\frac{1-\left(1+\frac{9,25}{1\,200}\right)^{-216}}{\frac{9,25}{1\,200}}$ $= \text{R}1\,555\,284,45$	interest rate for n Correct formula Substitution answer interest rate for n Correct formula Substitution answer

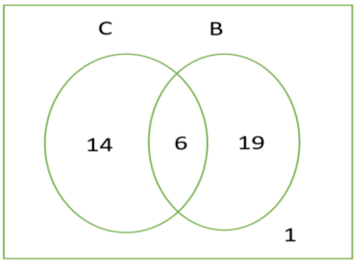
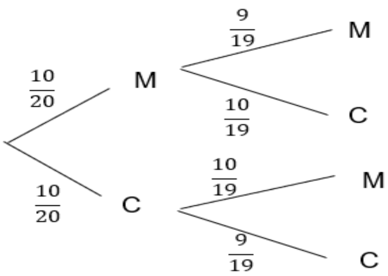
QUESTION 4

(a)(1)		Shape Vertical asymptote Horizontal asymptote *x*-intercept *y*-intercept
(a)(2)	$\mathbb{R}-\{-2\}$ **OR** $(-\infty\,;-2)\cup(-2\,;\infty)$ **OR** $\{x : x<-2 \text{ or } x>-2\}$	
(b)(1)	$y=3^x$ For inverse: let $x=3^y$ $g(x)=\log_3 x$	$x=3^y$ $g(x)=\log_3 x$
(b)(2)(i)	$x\in R$	$x\in R$
(b)(2)(ii)	$\therefore 0<x\leq 1$	

QUESTION 5

(a)	$f(x)=-2x^2+3x$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2(x+h)^2+3(x+h)-(-2x^2+3x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2x^2-4xh-2h^2+3x+3h+2x^2-3x}{h}$ $f'(x)=\lim_{h\to 0}\frac{-4xh-2h^2+3h}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-4x-2h+3)}{h}$ $f'(x)=\lim_{h\to 0}(-4x-2h+3)$ $=-4x+3$	$\frac{-2(x+h)^2+3(x+h)-(-2x^2+3x)}{h}$ Squaring and distributing Factorisation Notation Sub. to get: $=-4x+3$
(b)(1)	$y=2x^3-4x+5$ $\frac{dy}{dx}=6x^2-4$	$6x^2$ a -4
(b)(2)	$y=x^{-\frac{2}{5}}$ $\frac{dy}{dx}=-\frac{2}{5}x^{-\frac{7}{5}}$	$y=x^{-\frac{2}{5}}$ $\frac{dy}{dx}=-\frac{2}{5}x^{-\frac{7}{5}}$
(c)	For $f'(x)>0$ $f'(x)=-x^2+2x+3=0$ $x=3$ or $x=-1$ Increasing for: $-1\leq x\leq 3$ Also accept <	$f'(x)=-x^2+2x+3$ $f'(x)=0$ $x=3$ or $x=-1$ $-1\leq x\leq 3$

QUESTION 6

(a)(1)	No, There are 6 students playing both cricket and basketball.	No Explanation
(a)(2)	$P(C \text{ or } B) = P(C) + P(B) - P(C \text{ and } B)$ $P(C \text{ or } B) = \frac{20}{40} + \frac{25}{40} - \frac{6}{40}$ $= \frac{39}{40}$ Alternate:  $P(C \text{ or } B) = \frac{39}{40}$	$\frac{20}{40} + \frac{25}{40}$ $-\frac{6}{40}$ $= \frac{39}{40}$
(b)(1)		correct diagram correct M probabilities correct C probabilities
(b)(2)	P(M and C) OR P(C and M) $= \frac{10}{20} \times \frac{10}{19} + \frac{10}{20} \times \frac{10}{19}$ $= \frac{10}{19}$	$\frac{10}{20} \times \frac{10}{19}$ $\frac{10}{20} \times \frac{10}{19}$ $\frac{10}{19}$
(c)	$\frac{4!}{6!}$ $= \frac{4!}{6 \times 5 \times 4!}$ $= \frac{1}{30}$	$\frac{4!}{6!}$ $= \frac{4!}{6 \times 5 \times 4!}$ $= \frac{1}{30}$

SECTION B

QUESTION 7

(a)	$3500000 = P\left(1+\frac{4,5}{100}\right)^{13}$ $P = \text{R}1\ 974\ 950,74$	Correct formula $3500000 = P\left(1+\frac{4,5}{100}\right)^{13}$ $P = \text{R}1\ 974\ 950,74$
(b)	$1\ 800\ 000\left(1+\frac{5}{100}\right)^{n} = 1974\ 950,74\left(1+\frac{4,5}{100}\right)^{n}$ $\frac{\left(1+\frac{5}{100}\right)^{n}}{\left(1+\frac{4,5}{100}\right)^{n}} = \frac{1974\ 950,74}{1800\ 000}$ $\left(\frac{1,05}{1,045}\right)^{n} = 1,0971948...$ $n = 19,4$ Year: 2027	$1\ 800\ 000\left(1+\frac{5}{100}\right)^{n} = 1974\ 950,74\left(1+\frac{4,5}{100}\right)^{n}$ $\frac{\left(1+\frac{5}{100}\right)^{n}}{\left(1+\frac{4,5}{100}\right)^{n}} = \frac{1974\ 950,74}{1800\ 000}$ $n = 19,4$ 2027

QUESTION 8

(a)	$7 + x + y = -2x - 7$ $y = -3x - 14$ $\frac{x}{7} = \frac{-3x-14}{x}$ $x^2 + 21x + 98 = 0$ $x = -7$ or $x = -14$ $y = 7$ or $y = 28$	$7 + x + y = -2x - 7$ $\frac{x}{7} = \frac{-3x-14}{x}$ $x = -7$ or $x = -14$ $y = 7$ or $y = 28$
(b)	$S_n = \frac{n}{2}[2a + (n-1)d]$ $160 = \frac{n}{2}[2a + (n-1)(5)]$ $320 = 2an + 5n^2 - 5n$ $725 = \frac{2n}{2}[2a + (2n-1)(5)]$ $725 = 2an + 10n^2 - 5n$ $405 = 5n^2$ $n = 9$	$160 = \frac{n}{2}[2a + (n-1)(5)]$ $320 = 2an + 5n^2 - 5n$ $725 = \frac{2n}{2}[2a + (2n-1)(5)]$ $725 = \frac{2n}{2}[2a + (2n-1)(5)]$ $405 = 5n^2$ $n = 9$

QUESTION 9

(a)	$y=a(x+1)^2+q$ sub. $(-4;0)$ $0=a(-4+1)^2+q$ $0=9a+q$ .. eq 1 $y=a(x+1)^2+q$ sub. $(-2;4)$ $4=a(-2+1)^2+q$ $4=a+q$ ... eq.2 $a=-\frac{1}{2}$ $q=\frac{9}{2}$ $y=-\frac{1}{2}(x+1)^2+\frac{9}{2}$ $y=-\frac{1}{2}x^2-x+4$ $\therefore a=-\frac{1}{2}$; $b=-1$; $c=4$ **OR** x- intercepts are –4 and 2 $y=a(x+4)(x-2)$ Substitute (–2;4) $4=a(-2+4)(-2-2)$ $\therefore a=-\frac{1}{2}$ $y=-\frac{1}{2}(x+4)(x-2)$ $y=-\frac{1}{2}x^2-x+4$ $\therefore a=-\frac{1}{2}$; $b=-1$; $c=4$	$y=a(x+1)^2+q$ $0=9a+q$ .. eq 1 $4=a+q$ ... eq.2 $a=-\frac{1}{2}$ $b=-1$ $c=4$ $y=a(x+4)(x-2)$ Correct substitution $a=-\frac{1}{2}$ $b=-1$ $c=4$
(b)(1)	$f(x)=-\frac{1}{2}(x^2+2x-8)$ $f(x)=-\frac{1}{2}\left[(x+1)^2-8-1\right]$ $f(x)=-\frac{1}{2}\left[(x+1)^2-9\right]$ $f(x)=-\frac{1}{2}(x+1)^2+\frac{9}{2}$	$f(x)=-\frac{1}{2}(x^2+2x-8)$ $f(x)=-\frac{1}{2}\left[(x+1)^2-8-1\right]$ $f(x)=-\frac{1}{2}\left[(x+1)^2-9\right]$
(b)(2)	$-2<x<0$ and $x\neq-1$	$-2<x<0$ $x\neq-1$
(c)	X-int of g: $x=-8$ X-int of f: $x=2$ or $x=-4$ $x\in[-8;-4]\cup[2;\infty)$	$x\in[-8;-4]$ $[2;\infty)$

(d)	Top – Bottom $= x+8-\left[-\frac{1}{2}(x+1)^2+\frac{9}{2}\right]$ $= x+8+\frac{1}{2}x^2+x-4$ $=\frac{1}{2}x^2+2x+4$ $\frac{dy}{dx}=x+2$ $\therefore x=-2$ Top – Bottom $=\frac{1}{2}x^2+2x+4$ ... sub. $x=-2$ $\therefore$ Max. 2 units	$= x+8-\left[-\frac{1}{2}(x+1)^2+\frac{9}{2}\right]$ $=\frac{1}{2}x^2+2x+4$ $\frac{dy}{dx}=x+2$ $\therefore x=-2$ $\frac{1}{2}x^2+2x+4$ ...sub. $x=-2$ $\therefore$ Max. 2 units

QUESTION 10

$y=a(-x)^2-7(-x)+c$ $\therefore b=7$ $-\frac{b}{2a}=1$ $\therefore a=-\frac{7}{2}$ $f(x)=-\frac{7}{2}x^2+7x+c$ sub $(4;-20)$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$ $\therefore f(x)=-\frac{7}{2}x^2+7x+8$	$y=a(-x)^2-7(-x)+c$ $\therefore b=7$ $-\frac{b}{2a}=1$ $\therefore a=-\frac{7}{2}$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$
Alternative: $f(x)=a(-x)^2-7(-x)+c$ $f(x)=ax^2+7x+c$ $f'(x)=2ax+7$ for TP sub. $(x=1)$ $2a(1)+7=0$ $\therefore a=-\frac{7}{2}$ $\therefore b=7$ $f(x)=-\frac{7}{2}x^2+7x+c$ sub $(4;-20)$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$ $\therefore f(x)=-\frac{7}{2}x^2+7x+8$	$y=a(-x)^2-7(-x)+c$ $f'(x)=2ax+7$ $\therefore a=-\frac{7}{2}$ $\therefore b=7$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$

QUESTION 11

(a)	$-3^3+5\times3^2-3\times5+d=0$ $d=9$ **Alternate:** $y=-1(x-3)(x-3)(x+p)$ $y=-1(x^2-6x+9)(x+p)$ $y=-1(x^3-6x^2+9x+px^2-6px+9p)$ $y=-x^3+(6-p)x^2-(9-6p)x-9p$ $\therefore 9-6p=3$ $p=1$ $\therefore d=-9(p)=-9$	$-3^3+5\times3^2-3\times5+d=0$ $d=9$ $-1(x-3)(x-3)(x+p)$ $-x^3+(6-p)x^2-(9-6p)x-9p$ $\therefore 9-6p=3$ $p=1$ $\therefore d=-9(p)=-9$
(b)	$f'(x)=-3x^2+10x-3$ $f''(x)=-6x+10=0$ $-6x=-10$ $x=\frac{5}{3}$ Gradient = $-3\left(\frac{5}{3}\right)^2+10\times\frac{5}{3}-3$ $=\frac{16}{3}$	$f'(x)=-3x^2+10x-3$ $f''(x)=-6x+10=0$ $x=\frac{5}{3}$ $-3\left(\frac{5}{3}\right)^2+10\times\frac{5}{3}-3$ $=\frac{16}{3}$
(c)	$x<\frac{5}{3}$ Allow $x\leq\frac{5}{3}$	$x<\frac{5}{3}$
(d)	$f(x)-k+3=0$ $f(x)=k-3$ $f'(x)=-3x^2+10x-3=0$ $x=3$ or $x=\frac{1}{3}$ When $x=\frac{1}{3}$; $y=-\frac{256}{27}$ $\therefore -\frac{256}{27}\leq k-3\leq 0$ $\therefore -\frac{175}{27}\leq k\leq 3$	derivative =0 $x=3$ or $x=\frac{1}{3}$ substitution of one third $y=-\frac{256}{27}$ $\therefore -\frac{256}{27}\leq k-3\leq 0$ $\therefore -\frac{175}{27}\leq k\leq 3$

QUESTION 12

(a)	$f'(0)=15$	$f'(0)=15$
(b)	$f'(a)=0$ $\therefore a=10$	$\therefore a=10$
(c)	$m=\frac{15-0}{0-10}=-\frac{3}{2}$ $f'(x)=-\frac{3}{2}x+15$ $\therefore f'(1)=13{,}5$	for equation for gradient
(d)	Graph is concave down – no p.o.i	

QUESTION 13

(a)	Let: $x^2+2tx-t=x-\frac{5}{2}$ $x^2+(2t-1)x+\left(\frac{5}{2}-t\right)=0$ $\Delta=(2t-1)^2-4(1)\left(\frac{5}{2}-t\right)$ $4t^2-9<0$ Crit values: $-\frac{3}{2};\frac{3}{2}$ Will never touch for: $-\frac{3}{2}<t<\frac{3}{2}$	$x^2+2tx-t=x-\frac{5}{2}$ $\Delta=(2t-1)^2-4(1)\left(\frac{5}{2}-t\right)$ $\Delta<0$ or any method $4t^2-9<0$ Crit values: $-\frac{3}{2};\frac{3}{2}$ $-\frac{3}{2}<t<\frac{3}{2}$
(b)	$\frac{500}{x}-\frac{500}{x+3}=40$ $40x^2+120x-1500=0$ $2x^2+6x-75=0$ $x=4{,}8$ or $x=-7{,}8$ Original time: $\frac{500}{4{,}805}$ $=104{,}1$ seconds	$\frac{500}{x}-\frac{500}{x+3}=40$ $40x^2+120x-1500=0$ $x=4{,}8$ or $x=-7{,}8$ $\frac{500}{4{,}805}$ $=104{,}1$ seconds

Total: 150 marks