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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2021

MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. One blank page (page 24) has been included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure that your calculator is in **DEGREE** mode.
8. All the working details must be clearly shown. Answers only will not be awarded full marks.
9. Round off to one decimal place unless otherwise stated.
10. It is in your own interest to write legibly and to present your work neatly.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

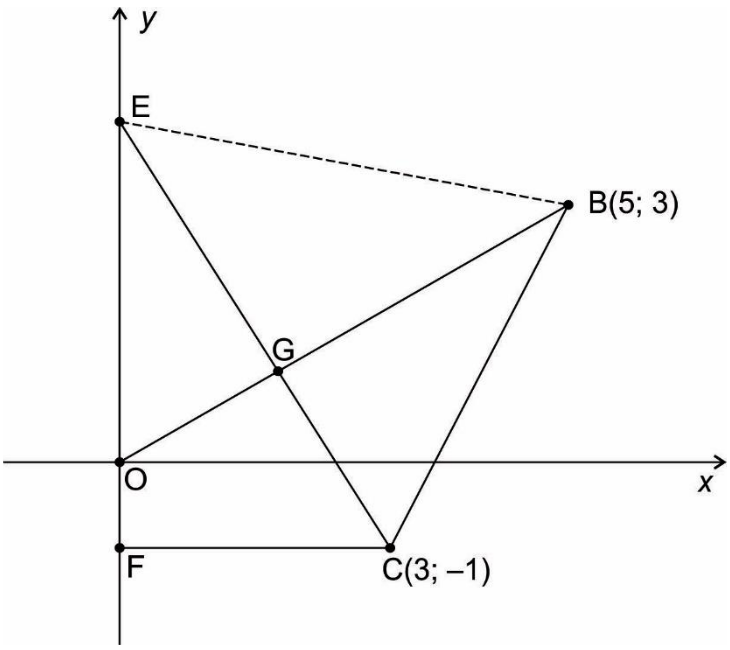
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
16	11	17	8	17	12	6	7	12	12	16	16	/150

SECTION A

QUESTION 1

In the diagram below:

- E, O and F lie on the *y*-axis
- The coordinates of B(5; 3) and C(3; –1) are given
- EC intersects OB at G
- FC is parallel to the *x*-axis



(a) Calculate the gradient of line OB.

(1)

(b) Determine the coordinates of E if EC is perpendicular to OB.

(3)

(c) Determine the length of straight line EB.

(2)

(d) Calculate the coordinates of point G.

(5)

(e) (1) Calculate the area of ΔEFC.

(2)

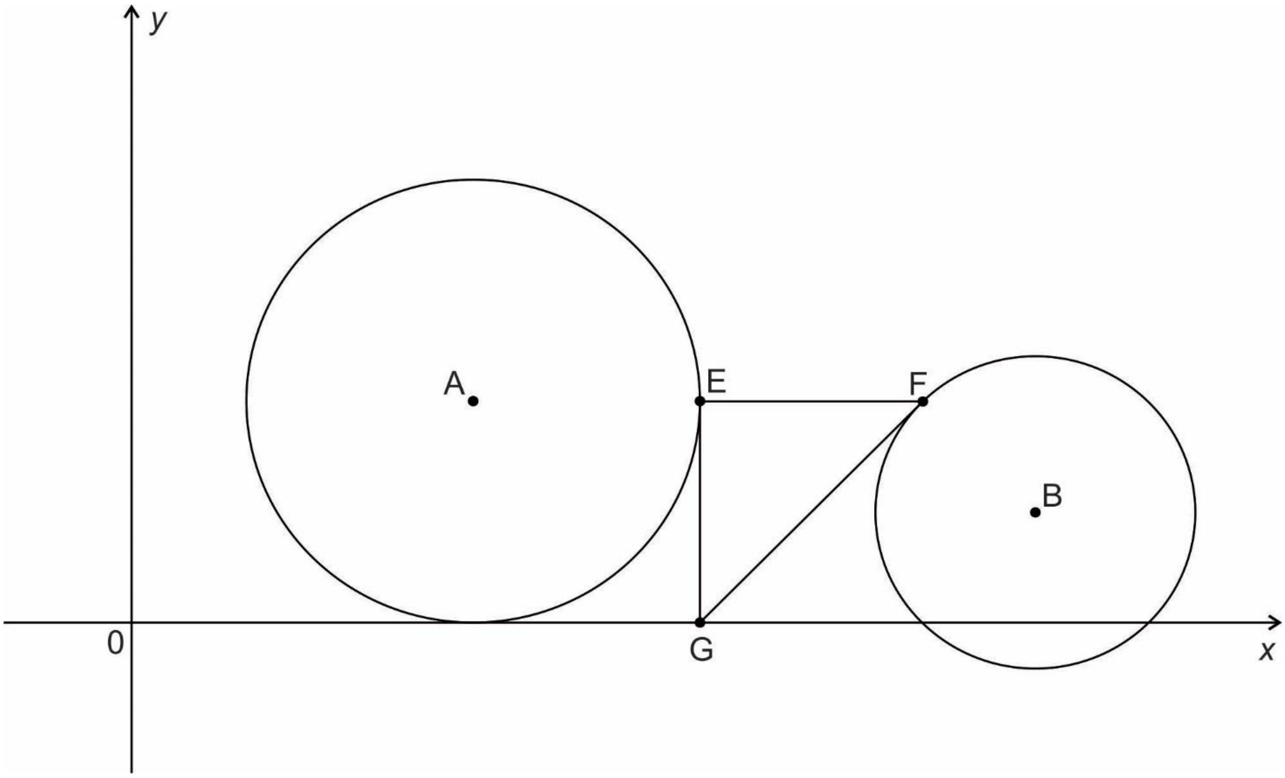
(2) Hence, or otherwise, calculate the area of the quadrilateral OGCF.

(3)
[16]

QUESTION 2

In the diagram below:

- E and F lie on the circles with centre A and B respectively and EF is parallel to the x-axis
- G lies on the x-axis with GE a tangent to circle A and also perpendicular to the x-axis
- GF is a tangent to circle B at F
- Equation of circle A: $(x-3)^2 + (y-2)^2 = 4$
- Equation of circle B: $x^2 - 16x + y^2 - 2y + 63 = 0$



(a) Determine the coordinates of point E.

(3)

(b) Determine the coordinates of B.

(2)

(c) Calculate the length of FG, leaving your answer in surd form.

(6)
[11]

QUESTION 3

(a) If $5\cos x = 4$ and $x \in [90°; 360°]$, determine the value of $3\sin 2x$ **without** using a calculator.

(5)

(b) Determine the general solution for x if $7 \sin 2x = \cos 2x$.

(5)

(c) (1) Given: $\dfrac{\cos 2\theta + 1}{\cos\theta} + 2\tan\theta.\sin\theta = \dfrac{A}{\cos\theta}$ where A is a constant.

Determine the value of A.

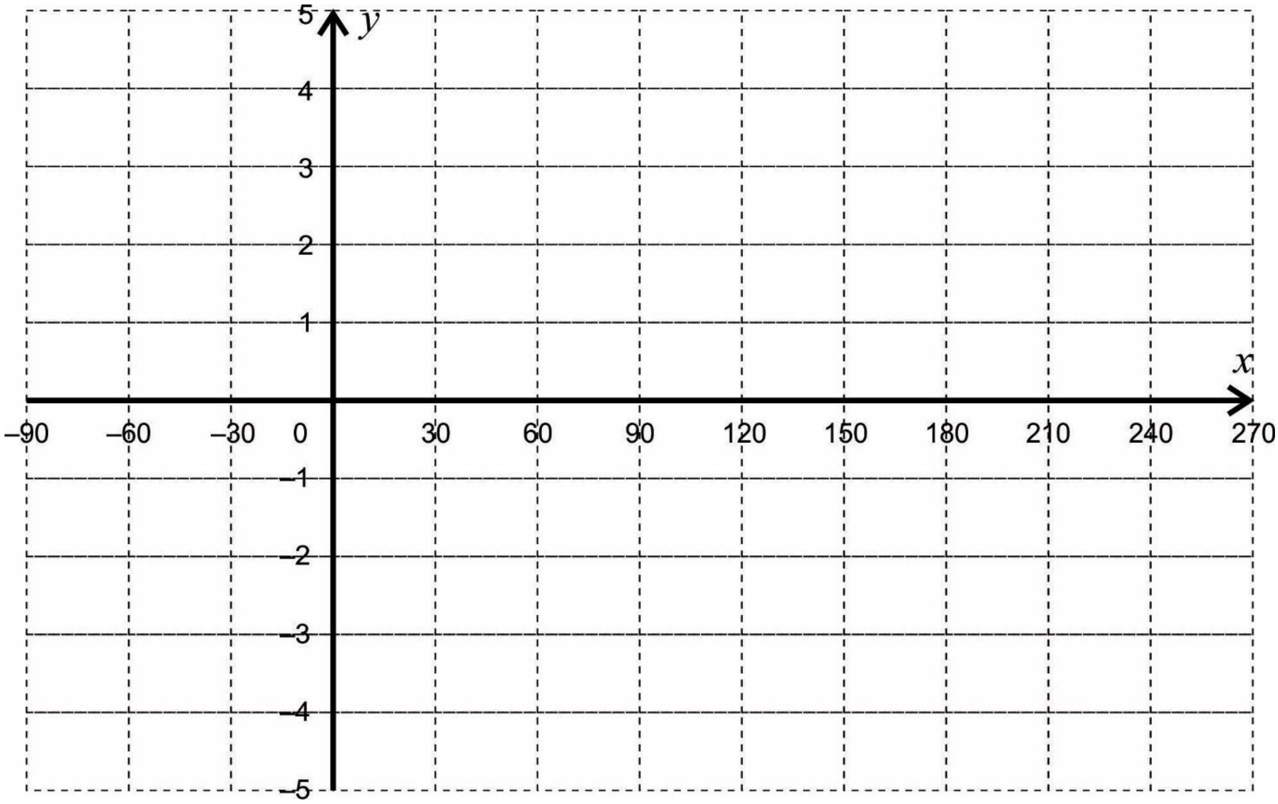
(4)

(2) Determine all values of θ for which the following expression is undefined: $\dfrac{\cos 2\theta + 1}{\cos\theta} + 2\tan\theta.\sin\theta$.

(3)
[17]

QUESTION 4

(a) On the set of axes below, sketch the graph of $f(x) = 3\sin(x - 60°)$ if $x \in [-90°; 270°]$. Show clearly coordinates of turning points, intercepts with axes and end-points.



(4)

(b) On the diagram above, indicate where $f(x) = -2$.

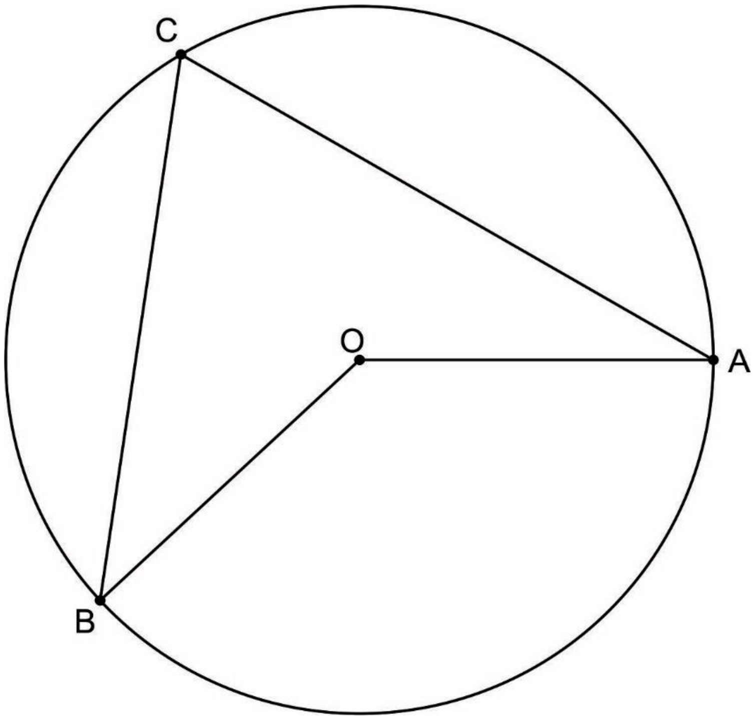
(2)

(c) For what values of k will $f(x) + k = 0$ have no real solutions?

(2)
[8]

QUESTION 5

(a) Prove the theorem that states: "The angle subtended by a chord at the centre of a circle is twice the size of the angle subtended by the same chord at a point on the circle in the same segment".



Required to prove: ___________________________

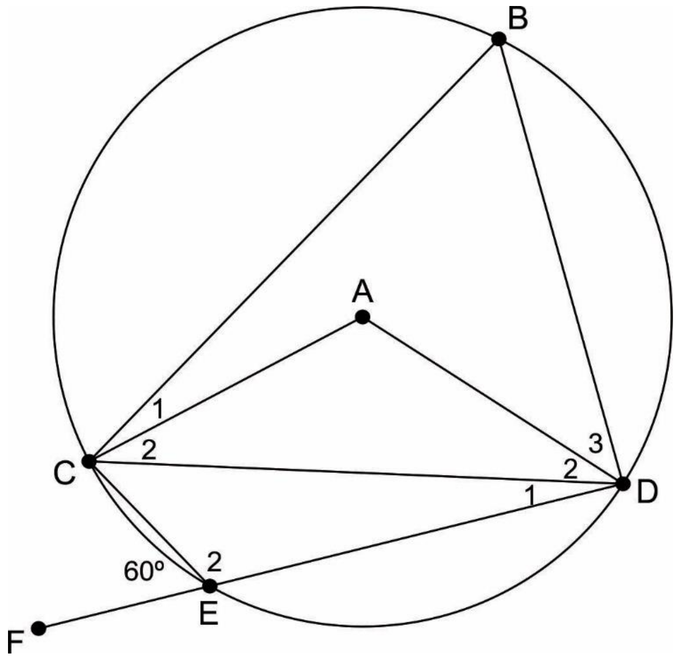
Construction: _______________________________

Proof:

(6)

(b) In the diagram below:

- A is the centre of the circle that goes through C, B, D and E
- FED is a straight line

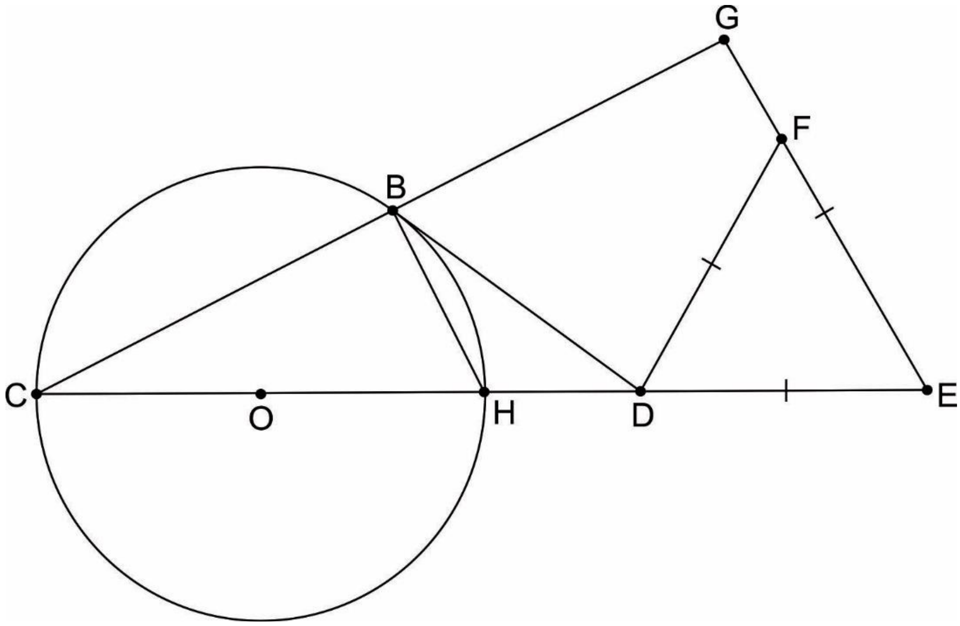


If $C\hat{E}F = 60°$ then determine the size of $\hat{C}_2$;

(6)

(c) In the diagram below:

- O is the centre of the circle passing through C, B and H
- CB and CH are extended to G and E respectively
- F and D are points on GE and CE respectively so that FD = DE = EF
- BD is a tangent to the circle at B
- $H\hat{B}D = 30°$



Prove that CE is the diameter of a circle that goes through C, G and E.

(5)
[17]

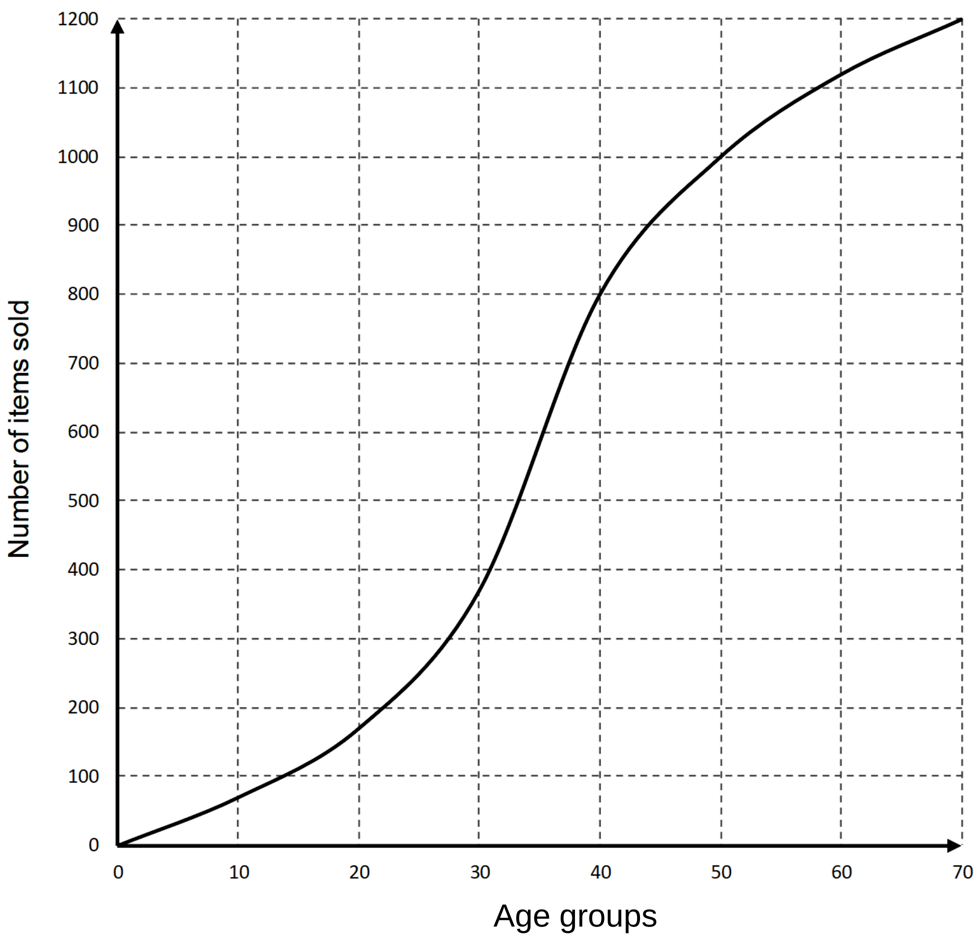
QUESTION 6

In the table below, a clothing shop has recorded the number of items that have been sold per age group.

Below is a cumulative frequency curve drawn from the values in the table below.

Use the diagram and table below to answer the questions that follow.

Age groups	Number of items sold	Cumulative frequency
$0 < x \leq 10$	70	70
$10 < x \leq 20$	100	170
$20 < x \leq 30$	200	370
$30 < x \leq 40$	**A**	**C**
$40 < x \leq 50$	**B**	**D**
$50 < x \leq 60$	120	1 120
$60 < x \leq 70$	80	1 200



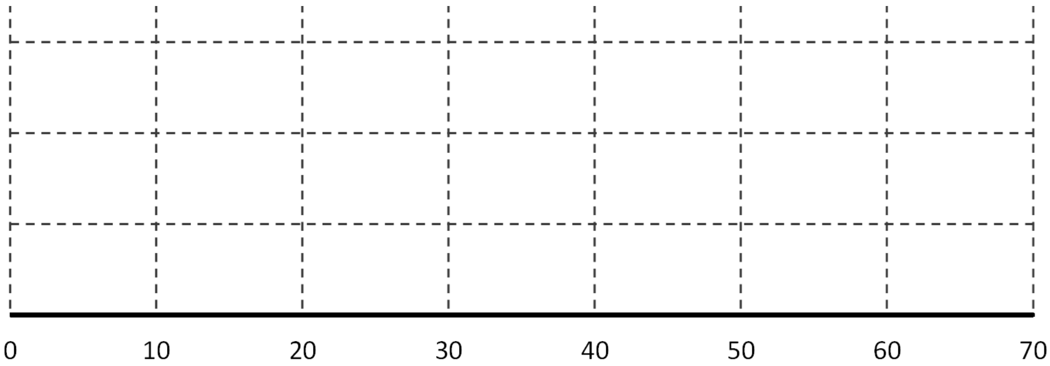
(a) What are the values of A, B, C and D in the table above?

A = __________ B = __________ C = __________ D = __________

(4)

(b) (1) Draw a box and whisker plot from the cumulative frequency curve shown in Question 6. Use the grid and information below.

- The lowest age recorded was three.
- The highest age recorded was 68.



(5)

(2) Is the data above skewed? Explain.

(2)

(3) The shop plans a sale on all items for children under the age of 12. Why will this skew the data?

(1)
[12]

81 marks

SECTION B

QUESTION 7

The table below gives statistical information about the buildings in a city.

Height of the buildings	
Mean	35 metres
Standard deviation	High

If a new building with a height of 35 metres was built in the city, then:

(a) How would this affect the mean? Explain.

(2)

(b) How would this affect the standard deviation? Explain.

(2)

(c) It is given that before the new building was built, the correlation coefficient for the height of buildings (x) and municipal electricity usage of buildings (y) was 0,92.

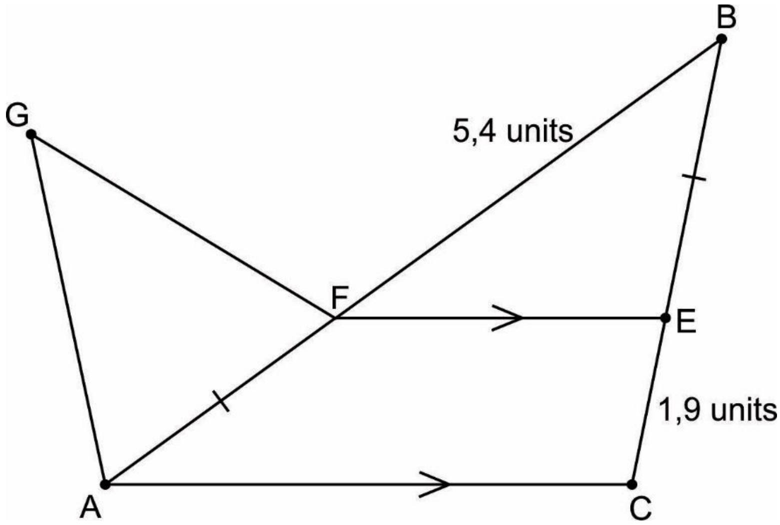
The new building uses solar power and NO electricity supplied by the municipality. How will this affect the correlation coefficient? Explain.

(2)
[6]

QUESTION 8

In the diagram:

- In ΔABC, F and E are points on AB and CB respectively
- FE || AC and BE = AF
- BF = 5,4 units
- EC = 1,9 units
- GA = GF = 4 units



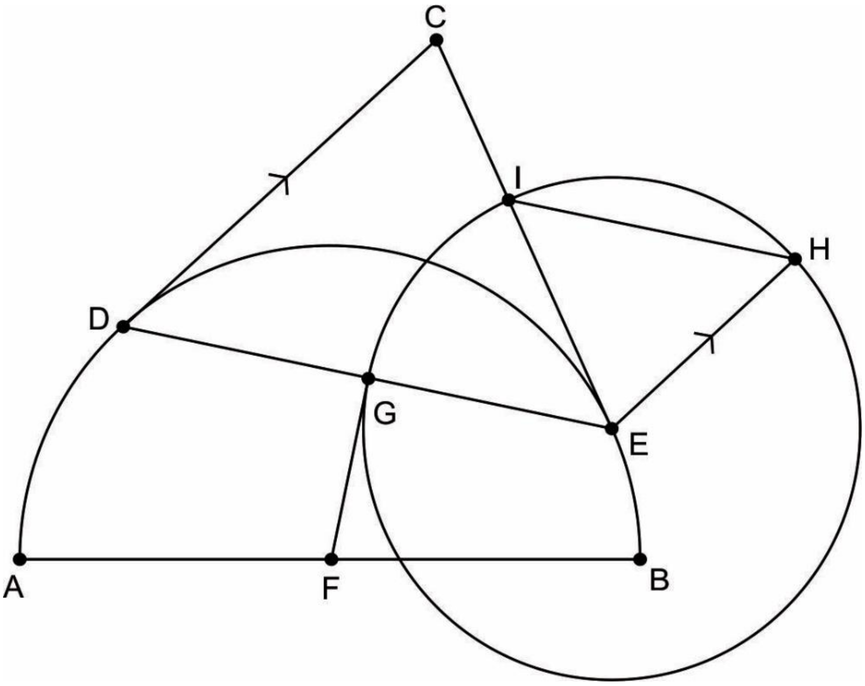
Calculate the size of $A\hat{G}F$.

[7]

QUESTION 9

In the diagram below:

- I, H and G are points on the circle with centre E
- AB is a diameter of the semi-circle with centre F, and D and E lie on the semi-circle
- CD and CE are tangents to the semi-circle at D and E respectively
- CD ll HE
- FG is a tangent to the circle with centre E



(a) Prove that ∆DCE lll ∆HEI.

(5)

(b) Prove that DG = GE.

(3)

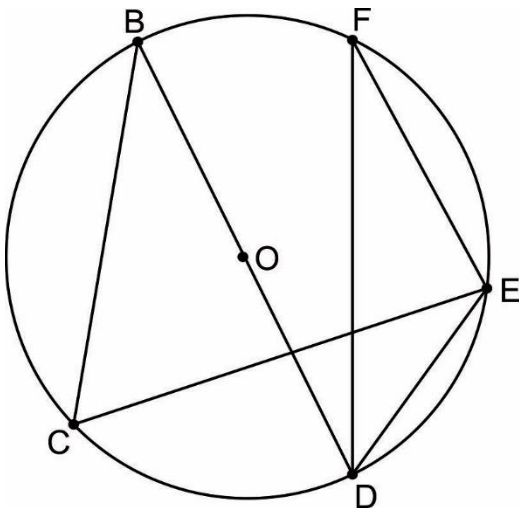
(c) Show that $2HE^2 = DC \times HI$.

(4)
[12]

QUESTION 10

(a) In the diagram below:

- O is the centre of the circle passing through B, C, D, E and F
- BD is a diameter
- $D\hat{F}E = x$ and $B\hat{C}E = 2x + 6°$

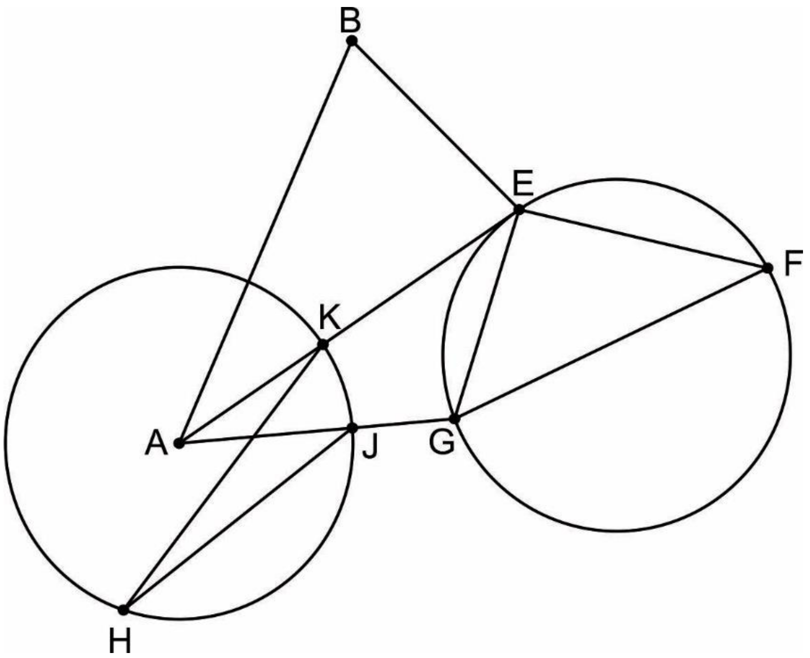


Determine the value of x.

(6)

(b) In the diagram below:

- H, J and K are points on a circle with centre A
- Another circle passes through G, E and F
- AE is a tangent at E and cuts the first circle at K
- $K\hat{H}J = x$; $E\hat{F}G = 55° - x$ and $A\hat{B}E = 55° + x$



Prove that ABEG is a cyclic quadrilateral.

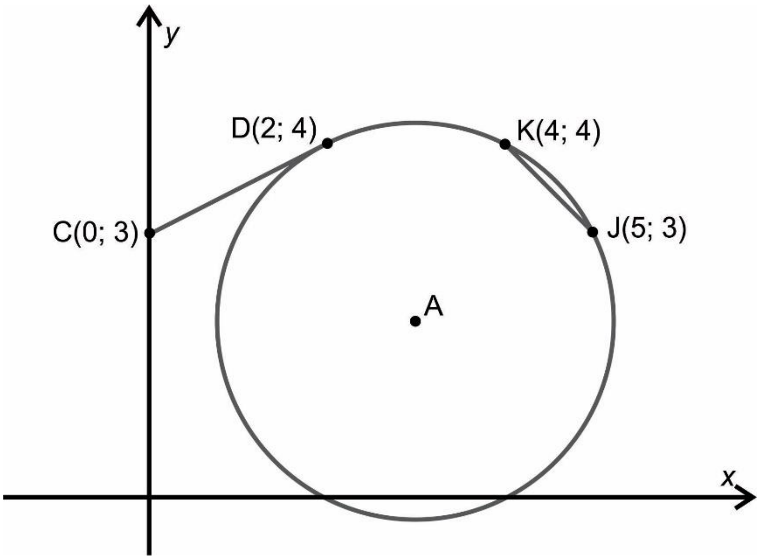
(6)

[12]

QUESTION 11

In the diagram:

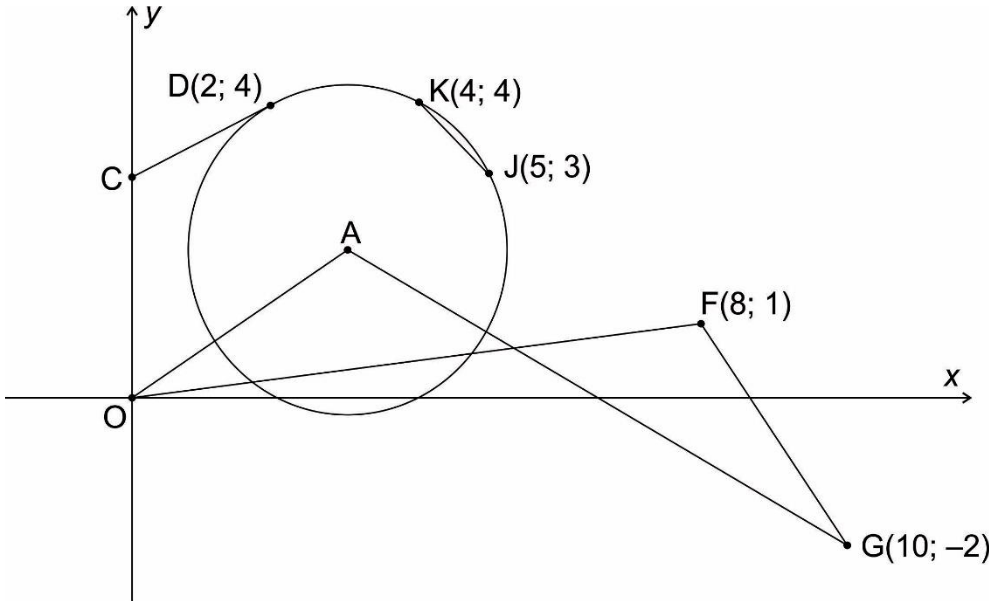
- The line from K(4; 4) to J(5; 3) is a chord of the circle with centre A
- CD is a tangent to the circle at D(2; 4)
- C(0; 3) lies on the *y*-axis



(a) Determine the coordinates of A (show all of your workings).

(8)

(b) Two additional points F(8;1) and G(10; –2) are added to the diagram with $A\hat{O}F = 26{,}6°$ and A(3;2).



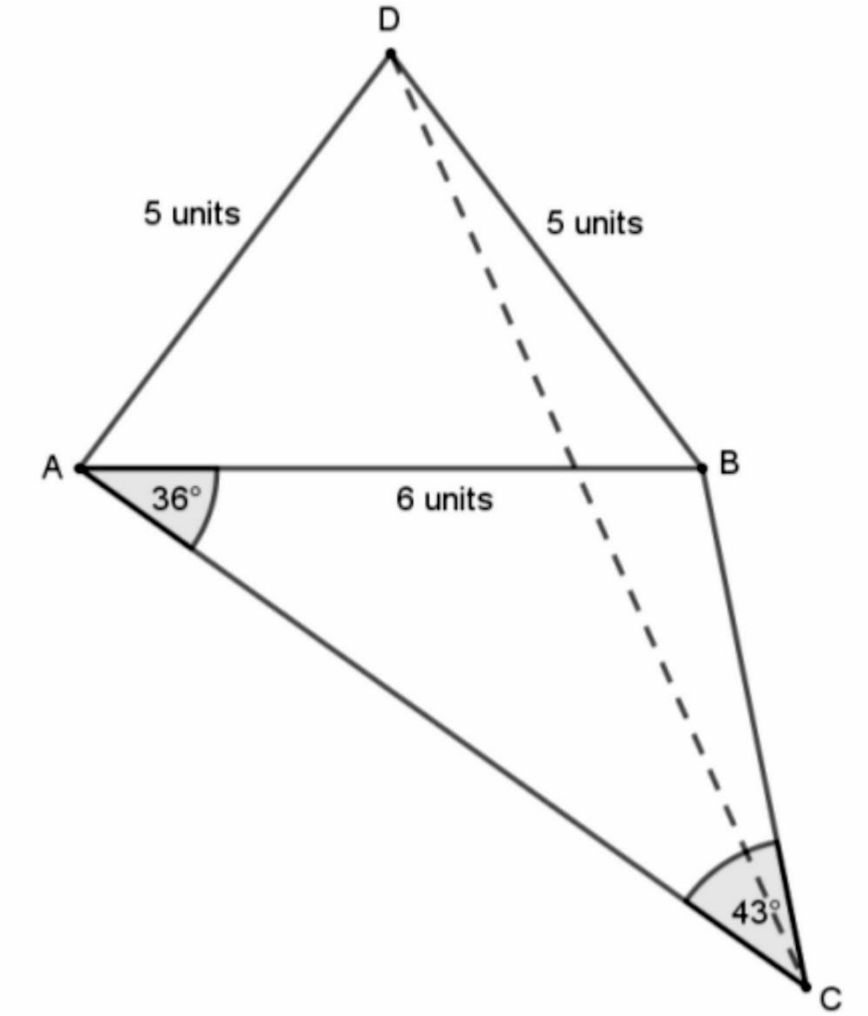
Prove that AFGO is a cyclic quadrilateral.

(8)
[16]

QUESTION 12

In the diagram below, $\triangle ABC$ lies on the horizontal plane and $\triangle ADB$ lies on the vertical plane.

- DB = DA = 5 units
- AB = 6 units
- $C\hat{A}B = 36°$ and $B\hat{C}A = 43°$



(a) Determine the vertical height of point D.

(3)

(b) Calculate the length of straight line DC, correct to two decimal digits.

(7)

(c) If ΔADB is folded back along line AB away from C so that D is on the same horizontal plane as A; B and C, then calculate the new straight-line distance from D to C.

(6)
[16]

69 marks

Total: 150 marks

ADDITIONAL SPACE TO ANSWER QUESTIONS. REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE ALL ANSWERS ARE MARKED.