assessment matters

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TOTAL
MARKS


NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2022

## MATHEMATICS: PAPER II

## EXAMINATION NUMBER



Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in DEGREE mode.
7. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers. Answers only will NOT necessarily be awarded full marks.
8. It is in your own interest to write legibly and to present your work neatly.
9. Round off to one decimal place unless otherwise stated.
10. Three blank pages (page 26-28) are included at the end of the question paper. If you run out of space for a question, use these pages. Clearly indicate the number of your answer, should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | TOTAL |
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| 17 | 12 | 13 | 21 | 7 | 8 | 12 | 7 | 7 | 24 | 15 | 7 | $/ 150$ |

## SECTION A

## QUESTION 1

A straight line goes through $P(2 ; 9)$ and $Q(6 ; 17)$.
(a) (1) Determine the gradient of line PQ.
(2) If $\mathrm{P}, \mathrm{Q}$ and $\mathrm{W}(-6 ; t)$ are collinear, then what is the value of $t$ ?
(3) Find the equation for the perpendicular bisector of $P Q$.
(A line perpendicular to PQ that goes through the midpoint of PQ.)
(4) If point $H(14 ; y)$ lies on the straight line with an equation of $4 y-x=2$; then what is the length of line PH ?
(4)
(b) In the diagram below, $A B$ has a gradient of 5 and the equation of $A C$ is $2 y-x+1=0$. Calculate the size of $\theta$.


## QUESTION 2

Given: $f(x)=3 \sin \left(x-45^{\circ}\right)$ and $g(x)=2$.
(a) Sketch the graphs of $f(x)$ and $g(x)$ on the set of axes below for $x \in\left[-45^{\circ} ; 360^{\circ}\right]$.

(b) Determine correct to one decimal place, the values of $x \in\left[-45^{\circ} ; 360^{\circ}\right]$ satisfying $f(x)=g(x)$.
(c) For what values of $x \in\left[-45^{\circ} ; 360^{\circ}\right]$ is $f(x)>g(x)$ ?

## QUESTION 3

(a) (1) Prove that $\frac{\cos 2 \theta+1}{\sin 2 \theta}+\tan \theta=\frac{1}{\sin \theta \cos \theta}$.
(2) Determine values for $\theta$ if $\theta \in\left[0^{\circ} ; 90^{\circ}\right]$ for which the identity is not valid.
(b) $\triangle A B C$ has side lengths $A B=3$ units, $B C=9$ units and $A C=10$ units.

(1) Calculate the size of $\hat{A}$.
(2) Calculate the area of $\triangle \mathrm{ABC}$.

## QUESTION 4

(a) Prove the theorem that states: 'The angle subtended by an arc at the centre of the circle is two times the angle that is subtended at the circumference by the same arc.'


Required to prove: $C O \hat{B}=2 C A \hat{B}$
Construction:
Proof:
(b) In the diagram below:

- Circle centre O is drawn.
- C, D, E, F and G lie on the circumference of the circle.
- $\mathrm{E} \hat{\mathrm{GF}}=36^{\circ} ; \mathrm{CEE}=50^{\circ} ; \mathrm{DCE}=22^{\circ} ; \mathrm{E} \hat{C} O=y$ and $\mathrm{OĈF}=x$.

(1) Determine the size of $y$, with reasons.
(2) Determine the size of $x$, with reasons.
(c) In the diagram below:
- Circle centre $O$ is drawn.
- $N, R, J, L$ and $K$ are points on the circle.
- LN and RK are diameters.
- LN intersects JK at M such that $\mathrm{JM}=\mathrm{MK}$.
- $\mathrm{JR}=\mathrm{RO}=5$ units.

(1) Determine, with reasons, the length of JK.
(2) Determine, with reasons, the length of ML.


## QUESTION 5

In the diagram below, $\triangle \mathrm{ABC}$ is drawn.

- $E$ is a point on $A C$ and $D$ is a point on $B C$ with $A B \| E D$.
- $F$ is a point on $E D$ and $G$ is a point on $E C$ with $F G \| B C$.
- $A E=4$ units; $B D=3$ units and $D C=9$ units.
- EF:FD=2:1.

(a) Calculate the length of EC.
(b) Calculate the length of FG.
(4)
[7]


## QUESTION 6

Refer to the cumulative frequency curve below that represents the results of 80 pupils that wrote a test out of 60 marks.

(a) How many pupils got between 12 and 48 for the test?
(b) If it is given that the lowest mark was 8 and the highest mark was 60 then sketch a box and whisker plot from the cumulative frequency curve on page 12.

(c) What percentage of pupils got more than $60 \%$ for the test?
(2)
[8]

78 marks

## SECTION B

## QUESTION 7

(a) For a set of data, will the data be skewed to the right or to the left if the median is significantly greater than the mean?
(b) The mean for a class test is $60 \%$. Will the standard deviation increase, decrease or stay the same if $2 \%$ is added to each of the learners' marks?
(c) If the estimated mean for the data below is 16,5 then what is the value of $\boldsymbol{k}$ ? (Show all of your workings.)

| Class Interval | Frequency |
| :---: | :---: |
| $0<x \leq 10$ | 9 |
| $10<x \leq 20$ | 16 |
| $20<x \leq 30$ | $\boldsymbol{k}$ |

(d) Refer to the scatter plot below that looks at the correlation between a student's Mathematics and Science results. Answer the questions that follow.

(1) Circle the correlation coefficient that best describes the data above.

A 0,2
B $\quad-0,93$
C 1
D 0,94
(2) The line of best fit for all these points has an equation of $y=A+B x$. Will the value of $B$ increase, decrease or stay the same if T is removed from the data? Explain.
(3) Given that $S$ is a point on the line of best fit, will the correlation co-efficient increase, decrease or stay the same if $S$ is removed? Explain.

## QUESTION 8

In the diagram below:

- Two circles that do not intersect are drawn.
- E; F and D lie on the larger circle.
- DC and EC are tangents to the larger circle at $D$ and $E$ respectively.
- The smaller circle is drawn so that MC is a chord.
- K and J lie on the smaller circle.
- $E$ F̂D $=x$.


Determine with reasons the size of MKJJ in terms of $x$.

## QUESTION 9

In the diagram below, a circle passing through F, E and D is drawn.

- $G$ is a point outside the circle so that $F G \| E D$ and $D G$ is a tangent to the circle at $D$.
- $H$ is a point inside the circle with $F \hat{H D}=90^{\circ}$.

(a) Prove that $\Delta F E D / / / \Delta G D F$.
(b) Show that $\mathrm{FH}^{2}+\mathrm{HD}^{2}=\mathrm{ED} . \mathrm{GF}$.



## QUESTION 10

(a) Given:

- $\sin \hat{A}=\frac{6}{10}$ and $90^{\circ}<\hat{A}<360^{\circ}$
- $\cos 42^{\circ}=p$

Without the use of a calculator, determine the value of $\cos \left(-A-42^{\circ}\right)$ in terms of $p$.
(b) Determine the general solution of the equation:
$4 \sin ^{2} \theta=\cos \left(90^{\circ}-2 \theta\right)$
(c) In the diagram below, quadrilateral CABD is drawn.

- AC//BD
- $\mathrm{BD}=p$ and $\mathrm{AC}=m$
- $\quad \mathrm{ABC}=\beta$ and $\mathrm{B} \hat{\mathrm{C}} \mathrm{A}=\theta$


Prove that: $\frac{m(\sin \beta \cos \theta+\sin \theta \cos \beta)}{\sin \beta}=\frac{p}{\cos \theta}$
(d) In the diagram below, $\mathrm{A}, \mathrm{C}$ and D lie in the same horizontal plane.

- $A B$ is a pole with its one end positioned at $A$.
- $B C$ and $B D$ are ropes used to hold the pole $A B$ up, with $C B D=30^{\circ}$.
- The length of rope $B C=$ the length of rope $B D$.
- $C D=3$ metres and $A D=4$ metres.


If the length of the pole $A B$ is 2 metres, then how much shorter should each rope be so that the pole, $A B$, is perpendicular to the ground?

## QUESTION 11

In the diagram below, circles with centres M and P touch each other externally at N .

- $M$, the centre of the bigger circle, lies on the $y$-axis.
- $P$, the centre of the smaller circle, lies on the $x$-axis.
- The equation of the straight line MNP is $3 y+4 x=12$.
- $\mathrm{MN}=3$ units.

(a) Determine the equation of the circle with centre $P$.
(b) Determine the $y$-intercept of the tangent common to the circles at N .
(8)


## QUESTION 12

In the diagram below:

- B is the centre of the circle $x^{2}-5 x+(y-3)^{2}=0$.
- $A B C$ is a diameter.
- $f(x)$ goes through the origin O and $\mathrm{C}(4 ; 1)$.


If $h(x)=f(x)+p x+t$, then find the values for $p$ and $t$ so that $h$ is a tangent at point C .

ADDITIONAL SPACE (ALL questions)
REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

