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NATIONAL SENIOR CERTIFICATE EXAMINATION MAY 2023

#### MATHEMATICS: PAPER II

#### MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

## NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

#### **SECTION A**

(a)	A = 14,533	A = 14,533
	<i>B</i> = 0,863	<i>B</i> = 0,863
	<i>y</i> = 14,533 + 0,863 <i>x</i>	<i>y</i> = 14,533 + 0,863 <i>x</i>
(b)(1)	<i>r</i> = 0,979	<i>r</i> ≈ 1
	<i>r</i> ≈ 1	
(b)(2)	There is a very strong positive correlation.	strong
		positive correlation
(c)	y = 14,533 + 0,863(70)	y = 14,533 + 0,863(70)
	<i>y</i> = 74,9	<i>y</i> = 74,9
	Alternate:	
	Using calculator: $y = 74,9$	
(d)	Interpolation usually results in a fairly reliable	fairly reliable
	prediction.	interpolation
	Alternates	
	Alternate:	
	$r \approx 1$ , hence strong correlation and reliable	
	prediction.	
	Alternate:	
	70 is not an outlier, hence a reliable prediction.	



(a)	$sin[(85^{\circ} + \theta) - (25^{\circ} + \theta)]$ sin(60^{\circ}) $= \frac{\sqrt{3}}{2}$	$\sin[(85^\circ + \theta) - (25^\circ + \theta)] = \frac{\sqrt{3}}{2}$
(b)	$-\tan(\theta).\cos(\theta) + \frac{2\sin\theta.\cos\theta}{2\cos(\theta)}$ $-\frac{\sin\theta}{\cos\theta}.\cos\theta + \frac{2\sin\theta.\cos\theta}{2\cos\theta}$ $-\sin\theta + \sin\theta$ $= 0$	$-\tan\theta$ $\cos\theta$ $2\sin\theta.\cos\theta$ $2\cos\theta$ $-\sin\theta + \sin\theta$ = 0
(c)(1)	$y^2 = 1^2 - p^2$ = $\sqrt{1 - p^2}$	$y^2 = 1^2 - p^2$ = $\sqrt{1 - p^2}$
(c)(2)	$=2\cos^2 x - 1$ $=2p^2 - 1$	$=2\cos^2 x-1$ $=2p^2-1$
(d)	$\frac{\sin(x-30^{\circ})}{\cos(x-30^{\circ})} = \frac{1}{2}$ $\tan(x-30^{\circ}) = \frac{1}{2}$ $x-30^{\circ} \approx 26, 6^{\circ} + k180^{\circ};  k \in \mathbb{Z}$ $x \approx 56, 6^{\circ} + k180^{\circ}  ;  k \in \mathbb{Z}$	$ \tan(x-30^\circ) = \frac{1}{2} $ Ref angle: 26,6° $x \approx 56,6^\circ + k180^\circ; k \in Z$



(a)	$\hat{A}_1 = A\hat{D}O$ (Radii/ $\angle$ s opp. = sides)	$\hat{A}_1 = A\hat{D}O$
	$\therefore \hat{A}_1 = \frac{180^\circ - 40^\circ}{2}  (\text{int. } \angle \text{s of } \Delta)$	(Radii/∠s opp. = sides)
	$\therefore \hat{A}_1 = 70^{\circ}$	$\therefore A_1 = 70^\circ$ (int. $\angle$ s of $\Delta$ )
(b)	$\hat{E} = 110^{\circ}$ (Opp. $\angle$ s of cyclic quad.)	$\hat{E} = 110^{\circ}$ (Opp. $\angle$ s of cyclic quad.)
	Alternate	
	$\hat{O}_2 = 180^\circ - 40^\circ$ (Adj. $\angle$ s on str line)	$\hat{O}_2 = 140^\circ$
	$\hat{O}_2 = 140^\circ$	$\dot{\mathbf{C}}_2 = 70^{\circ}$
	$\therefore \hat{C}_2 = 70^\circ$ ( $\angle$ at centre $= 2 \times \angle$ at circumf.)	$(\angle \text{ at centre } = 2 \times \angle \text{ at circumf.})$
	$\hat{E} = 110^{\circ}$ (Opp. $\angle$ s of cyclic guad.)	E = 110°
		(Opp. ∠s of cyclic quad.)
	Alternate:	
	Reflex. $DOB = (O_1 + 180^\circ)$ (adj. $\angle$ on str. line) Reflex. $DOB = 220^\circ$	Reflex. $DOB = 220^{\circ}$
	$\hat{E} = 110^{\circ}$ ( $\angle$ at centre = 2× $\angle$ at circumf.)	Ê = 110°
		$(\angle \text{ at centre } = 2 \times \angle \text{ at circumf.})$
(c)	$  \hat{C}_1 = 20^{\circ}  (\angle \text{ at centre } = 2 \times \angle \text{ at circumf.} ) $	$ \hat{\hat{C}}_1 = 20^{\circ}  (\angle \text{ at centre } = 2 \times \angle \text{ at circumf.} ) $
	Alternate:	
	$\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2 = 90^{\circ}$	
	$(\angle$ in semi-circle)	
	and $\therefore \hat{C}_2 = 70^\circ$ (proven)	
	$\therefore \hat{C}_1 = 20^{\circ}$	
(d)	$\hat{B}_1 = 40^{\circ}$	$\hat{B}_1 = 40^{\circ}$
	(corresp ∠s DO//EB)	(corresp ∠s DO//EB)

(e)	AF = FE (line from midpoint // to one side) DO $\perp$ AE (line from centre to midpoint of	AF = FE (line from midpoint // to one side)
	chord) In $\triangle AOF$ : sin 40° = $\frac{AF}{4\frac{1}{4}}$	DO ⊥ AE (line from centre to midpoint of chord)
	2 AF ≈ 2,9 units ∴ FE ≈ 2,9 units	In $\triangle AOF$ : sin 40° = $\frac{AF}{4\frac{1}{2}}$
	$\Delta E \sim 5.8$ upito	AF ≈ 2,9 units ∴ FE ≈ 2,9 units
	Alternate:	∴ AE ≈ 5,8 units
	In In $\triangle ABE$ : $\hat{E} = 90^{\circ}$ ( $\angle$ in semi-circle) Hence: $\sin 40^{\circ} = \frac{AE}{9}$	
	∴ AE ≈ 5,8 units	

(a)	$\hat{A} = x$ (tap/cbord theorem)	$\hat{A} = x$ (tap/cbord theorem)
(b)	In $\triangle$ CBD and $\triangle$ ACD: $\hat{B}_2 = 90^\circ$ (adj. $\angle$ s on str line)	$\hat{C}_2 = \hat{A}$ (tan/chord theorem)
	$\hat{ACD} = 90^{\circ}$ (tan $\perp$ rad)	$\hat{D}$ is common
	$\hat{C}_2 = \hat{A}$ (tan/chord theorem)	∴∆CBD /// ∆ACD (∠∠∠)
	Ď is common ∴ ΔCBD /// ΔACD (∠∠∠)	
(c)	$\frac{CD}{BD} = \frac{AD}{CD}  (///\Delta s - side in prop)$	$\frac{CD}{BD} = \frac{AD}{CD}$
	$CD^2 = 9 \times 4$	(/// $\Delta$ s - side in prop)
	$\therefore$ CD = 6 units	$CD^2 = 9 \times 4$
		$\therefore CD = 6$

(a)	$\hat{B} = 90^{\circ} (\angle \text{ in semi } \odot)$	$\hat{B} = 90^{\circ} (\angle \text{ in semi } \odot)$
	$\hat{C}_1 = 45^\circ$ (given)	$\therefore \hat{A}_1 = 180^\circ - (90^\circ + 45^\circ)$
	$\therefore \stackrel{\circ}{A}_1 = 180^\circ - (90^\circ + 45^\circ)$ (int. $\angle$ s of $\triangle$ )	(int. $\angle$ s of $\Delta$ )
	$\therefore \hat{A}_1 = 45^\circ$ (isosceles $\Delta$ / sides opp = $\angle$ s)	$\therefore \hat{A}_1 = 45^\circ$
		(isos $\Delta$ / sides opp = $\angle$ s)
(b)	$\overset{\circ}{B}_2 = 90^\circ - 67,5^\circ \ (\angle \text{ in semi } \odot)$	$\hat{B}_2 = 90^\circ - 67,5^\circ$
	$\hat{B}_2 = 22,5^{\circ}$	(∠ in semi ⊙)
	$\therefore \hat{A}_2 = 22,5^\circ \ (\angle \text{ in same seg})$	$\therefore \hat{A}_2 = 22,5^\circ$
	$\hat{A}_1 = 45^\circ$ (shown)	(∠ in same seg)
	$\therefore \hat{A}_1 = 2 \times \hat{A}_2$	$\hat{A}_1 = 45^\circ$ (shown)

# **SECTION B**

(a)	<b>Construction:</b> Join AO and BO In $\triangle AOC$ and $\triangle BOC$ OC is a common side AO = BO (radii) $\hat{C}_1 = \hat{C}_2 = 90^\circ$ (given) $\therefore \triangle AOC \equiv \triangle BOC$ (R;H;S) Hence $AC = CB$	Join AO and BO OC is a common side AO = BO (radii) $\hat{C}_1 = \hat{C}_2 = 90^\circ$ (given) $\therefore \Delta AOC \equiv \Delta BOC$ (R;H;S)
(b)	Let: line perp to AB meet AB at M $\therefore$ CM goes through the centre BM = MA = 4 units (line from centre perp to chord) In $\triangle$ AOM: Let OM = x $\therefore$ radius CO = 8 - x $\therefore (8 - x)^2 = x^2 + 4^2$ (pythag) 16x = 48 x = 3 $\therefore$ radius is 5 units Alternate: Let the radius be r In $\triangle$ AOM: OM = 8 - r $\therefore r^2 = (8 - r)^2 + 4^2$	BM = MA = 4 units (line from centre perp to chord) ∴ radius CO = 8 - x ∴ $(8 - x)^2 = x^2 + 4^2$ (pythag) x = 3 ∴ radius is 5 units
	$\therefore 16r = 80$ Hence $r = 5$	

(a)	In $\triangle ACG: \frac{AE}{EC} = \frac{AF}{FG}$ (line    one side of $\triangle$ )	AE AF
	$\frac{3p}{2p} = \frac{2k}{FG}$	$\overline{EC} = \overline{FG}$ (line    one side of $\Delta$ )
	$\therefore FG = \frac{4}{3}k$	$\therefore FG = \frac{4}{3}k$
	In $\triangle BFD$ : $\frac{BG}{GF} = \frac{BC}{CD}$ (line    one side of $\triangle$ ) $\frac{\frac{11}{3}k}{\frac{4}{6}k} = \frac{BC}{CD}$	$\frac{\frac{11}{3}k}{\frac{4}{3}k} = \frac{BC}{CD}$ (line    one side of $\Delta$ )
	$\frac{3}{BC} = \frac{11}{4}$	$\frac{BC}{CD} = \frac{11}{4}$

(a)(1)	$\frac{\sin\theta.\cos 2\theta}{2\sin\theta\cos\theta} \div \left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\right)$ $= \frac{\sin\theta.\cos 2\theta}{2\sin\theta\cos\theta} \div \left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\right)$ $= \frac{\sin\theta.\cos 2\theta}{2\sin\theta\cos\theta} \times \left(\frac{\sin\theta\cos\theta}{-\cos 2\theta}\right)$ $a f(\theta) = -\frac{1}{2}\sin\theta$	$2\sin\theta\cos\theta$ $\frac{\sin\theta}{\cos\theta}$ $\left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\right) \text{ for LCD}$ $-\cos2\theta$ $= -\frac{1}{2}\sin\theta$
(a)(2)	Values of $\theta \in [0^{\circ}; 360^{\circ}]$ for which the identity is not valid: tan $\theta$ is undefined for: $\{90^{\circ}; 270^{\circ}\}$ sin $2\theta = 0$ undefined for: $\{0^{\circ}; 180^{\circ}; 360^{\circ}\}$ tan $\theta - \frac{1}{\tan \theta} = 0$ undefined for: $\{45^{\circ}; 135^{\circ}; 225^{\circ}; 315^{\circ}\}$	$\{90^{\circ};270^{\circ}\}$ $sin2\theta = 0$ $\{0^{\circ};180^{\circ};360^{\circ}\}$ $tan\theta - \frac{1}{tan\theta} = 0$ $\{45^{\circ};135^{\circ};225^{\circ};315^{\circ}\}$
(b)	$A\hat{D}C = 180^{\circ} - (\alpha + \beta) \text{ (int. } \angle \text{s of } \Delta)$ $\frac{AC}{\sin A\hat{D}C} = \frac{AD}{\sin \beta}  \text{(sine rule)}$ $\frac{120}{\sin [180^{\circ} - (\alpha + \beta)]} = \frac{AD}{\sin \beta}$ $\therefore AD = \frac{120 \sin \beta}{\sin (\alpha + \beta)}$ $In \ \Delta ABD: \ \tan \theta = \frac{BD}{AD}$ $BD = AD \tan \theta$ $\therefore BD = \frac{120 \sin \beta \cdot \tan \theta}{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}$	$A\hat{D}C = 180^{\circ} - (\alpha + \beta)$ $\frac{120}{\sin[180^{\circ} - (\alpha + \beta)]} = \frac{AD}{\sin\beta}$ $\therefore AD = \frac{120\sin\beta}{\sin(\alpha + \beta)}$ $In \ \Delta ABD: \ \tan\theta = \frac{BD}{AD}$ $BD = AD\tan\theta$

(a)	$(x+4)^2 + (y-5)^2 = r^2$	$(x+4)^2 + (y-5)^2 = r^2$
	$F\left(\frac{-4-2}{2};\frac{5-1}{2}\right)$	$F\left(\frac{-4-2}{2};\frac{5-1}{2}\right)$
	$\therefore$ F(-3;2) sub. in eq of $\odot$	$(-3+4)^2 + (2-5)^2 = r^2$
	$(-3+4)^2 + (2-5)^2 = r^2$	$r^2 = 10$
	$r^2 = 10$	$(x+4)^2+(y-5)^2=10$
	Eq.: $(x+4)^2 + (y-5)^2 = 10$	
(b)	$m_{AB} = -\frac{1}{2}$	$m_{AB} = -\frac{1}{2}$
	$m_{\scriptscriptstyle BC}=2$	$m_{\scriptscriptstyle BC}=2$
	Eq. BC: $y = 2x + c$ sub. (-10;3)	$\therefore c = 23$
	$\therefore c = 23$	$m_{12} = -3$
	Eq. BC: $y = 2x + 23$	$\therefore c = -7$
	$m_{AC} = -3$	
	Eq. AC: $y = -3x + c$ sub. $(-2; -1)$	x = -6
	$\therefore c = -7$	$\therefore y = 11$
	Eq. AC: $y = -3x - 7$	C(-6;11)
	For C: $-3x-7 = 2x+23$	
	x = -6	
	$\therefore y = 11$ C(6:11)	
(c)	D = (-0, 11)	DE   CE (tan   rad)
(0)	$DE = \sqrt{10}$	$CD = 2\sqrt{10}$
	$CD = \sqrt{(-4+6)^2 + (5-11)^2}$	$CF^{2} - (2\sqrt{10})^{2} - (\sqrt{10})^{2}$
	$CD = \sqrt{(-4+0)^2 + (3-11)^2}$	CE = (2410) - (410)
	$CD = 2\sqrt{10}$	$CE = \sqrt{30}$
	$CE^{2} = (2\sqrt{10})^{2} - (\sqrt{10})^{2}$ (pythag)	
	$CE = \sqrt{30}$	

(-)	2 2	<u> </u>
(a)	$(x-7)^{2} + (y-1)^{2} = -46 + 49 + 1$	$(x-7)^2 + (y-1)^2 =$
	$(x-7)^{2} + (y-1)^{2} = 4$	-46+49+1
	Centre: (7;1)	Centre: (7;1)
	Radius: 2 units	Radius: 2 units
(b)	$m_{PA} = -\sqrt{3}$	$m_{_{PA}} = -\sqrt{3}$
	$\therefore m_{\rm tan} = \frac{1}{\sqrt{3}}$	$\therefore m_{\rm tan} = \frac{1}{\sqrt{3}}$
	$y = \frac{1}{\sqrt{2}}x + c$ sub (6; $\sqrt{3} + 1$ )	$c=1-\sqrt{3}$
	$c = 1 - \sqrt{3}$	$y = \frac{1}{\sqrt{3}}x + 1 - \sqrt{3}$
	$y = \frac{1}{\sqrt{3}}x + 1 - \sqrt{3}$	
(c)(1)	$(x-7)^{2}+(y-1)^{2}=4$	LHS = $(x-7)^{2} + (y-1)^{2}$
	LHS = $(x-7)^2 + (y-1)^2$ sub $(8; -\sqrt{3}+1)$	sub $(8; -\sqrt{3}+1)$
	LHS = $(8-7)^{2} + (-\sqrt{3}+1-1)^{2}$	
	LHS = 4	LHS = $(8-7)^2 + (-\sqrt{3}+1-1)^2$
	$LHS = RHS$ $\therefore$ point lies on the circle	LHS = 4
		$\therefore$ point lies on the circle
(c)(2)	Dist AB $-\sqrt{(8-6)^2 + (-\sqrt{3}+1-\sqrt{3}-1)^2}$	Dist AB =
	Dist AB = $\sqrt{(0-0)^{-1}(-\sqrt{3}+1-\sqrt{3}-1)}$ Dist AB = 4	$\sqrt{(8-6)^2 + (-\sqrt{3}+1-\sqrt{3}-1)^2}$
	This is twice the redius, therefore AD is a	Dist AB = 4
	diameter.	This is twice the radius, therefore AB is a diameter.
1	1	

(2)		^ <b>1</b>
(a)	$\Delta BEM \equiv \Delta CEM$ (R;H;S)	$\tan \hat{E}_1 = \frac{4}{6}$
	$\therefore$ BM = MC = 4 cm	$\hat{E}_1 = 33,7^{\circ}$
	$\tan E_1 = \frac{4}{6}$	∴BÊC=67,4°
	$\hat{E}_1 = 33,7^{\circ}$	
	$\therefore \hat{BEC} = 67,4^{\circ}$ Snow will build up on the roof.	
	Alternate: In $\triangle BCE$ : $BE^{2} = 4^{2} + 6^{2}$ (pythag) $BE = 2\sqrt{13}$ Cosine Rule: $\hat{A} = \cos^{-1} \left( \frac{8^{2} - (2\sqrt{13})^{2} - (2\sqrt{13})^{2}}{-2(2\sqrt{13})(2\sqrt{13})} \right)$	
	= 67,4°	
(b)	DM <sup>2</sup> = 12 <sup>2</sup> + 4 <sup>2</sup> (pythag) DM = 4 $\sqrt{10}$ In $\Delta$ EDM: DE <sup>2</sup> = 6 <sup>2</sup> + $(4\sqrt{10})^2$ DE = 14 cm sinDÊC = $\frac{12}{14}$ DÊC = 59°	$DM = 4\sqrt{10}$ $\Delta EDM: DE^{2} = 6^{2} + (4\sqrt{10})^{2}$ DE = 14  cm $\sin D\hat{E}C = \frac{12}{14}$ $D\hat{E}C = 59^{\circ}$
	Alternate: In $\triangle$ MEC: CE <sup>2</sup> = 4 <sup>2</sup> + 6 <sup>2</sup> (pythag) CE = 2 $\sqrt{13}$	
	DC = 12 cm DE <sup>2</sup> = 12 <sup>2</sup> + $(2\sqrt{13})^2$ DE = 14 cm sinD $\hat{E}C = \frac{12}{14}$ D $\hat{E}C = 59^\circ$	

(a)	B(x;-2x)	B(x,-2x)
	Dist OB = $\sqrt{(x-0)^2 + (-2x-0)^2}$	$\sqrt{\left(x-0\right)^2 + \left(-2x-0\right)^2} = \sqrt{125}$
	$\sqrt{(x-0)^2 + (-2x-0)^2} = \sqrt{125}$	$x = \pm 5$
	$5x^2 = 125$	
	$x = \pm 5$ B(-5; v) sub x = -5 in f(x)	
	∴B(-5;10)	
(b)	$m_{OB} = -2$	$m_{\rm OB} = -2$
	$\therefore m_{BE} = \frac{1}{2}$	$\therefore m_{BE} = \frac{1}{2}$
	Eq BE: $y = \frac{1}{2}x + c$ sub (-5;10)	$c=\frac{25}{2}$
	$c=\frac{25}{2}$	25
	$\therefore E\left(0;12\frac{1}{2}\right)$	$\frac{1}{8}$ 25
		$\therefore y = \frac{23}{4}$
	$m_{EC} = -2$ (// lines)	25√5
	Eq EC: $y = -2x + \frac{25}{2}$	Dist EC = $\frac{1000}{8}$
	For C: $-2x + \frac{25}{2} = 2x$	Dist BE = $\frac{5\sqrt{5}}{2}$
	$x = \frac{25}{9}$	
	$\therefore y = \frac{25}{4}$	$=\frac{1}{2}\times\left(\sqrt{125}+\frac{23\sqrt{3}}{8}\right)\times\frac{3\sqrt{3}}{2}$
	$C\left(\frac{25}{25};\frac{25}{25}\right)$	≈ 50,8 units²
	Dist EC = $\sqrt{\left(\frac{25}{8} - 0\right)^2 + \left(\frac{25}{4} - \frac{25}{2}\right)^2}$	
	Dist EC = $\frac{25\sqrt{5}}{8}$	
	Dist BE = $\sqrt{(-5-0)^2 + (10-\frac{25}{2})^2}$	
	Dist BE = $\frac{5\sqrt{5}}{2}$	

Area of Trap = $\frac{1}{2} \times \left( \sqrt{125} + \frac{25\sqrt{5}}{8} \right) \times \frac{5\sqrt{5}}{2}$	
Area of Trap = $\frac{1625}{32}$	
$\approx 50,8 \text{ units}^2$	
Alternate:	
In $\triangle BOE$ : $m_{OB} = -2$ $\therefore \tan \theta = 2$ $\therefore \theta = 63, 4^{\circ}$ $\therefore BOE = 90^{\circ} - 63, 4^{\circ}$ $\therefore BOE = 26, 6^{\circ}$	
Eq BE: $y = \frac{1}{2}x + c$ sub (-5;10) $c = \frac{25}{2}$ $\therefore E\left(0;12\frac{1}{2}\right)$	
$\therefore EO = 12\frac{1}{2} \text{ units and OB} = \sqrt{125}$ Area $\triangle BOE = \frac{1}{2} \left( 12\frac{1}{2} \right) \left( \sqrt{125} \right) \sin 26, 6^{\circ}$ Area $\triangle BOE = 31,2881 \text{ units}^2$	
In ∆EOC: CÉO=26,6° (alt. ∠s, BO//EC)	
$m_{EC} = -2$ (// lines) Eq EC: $y = -2x + \frac{25}{2}$	
For C: $-2x + \frac{25}{2} = 2x$ $x = \frac{25}{8}$ $\therefore y = \frac{25}{4}$ $C\left(\frac{25}{8}; \frac{25}{4}\right)$	

Dist EC = $\sqrt{\left(\frac{25}{8} - 0\right)^2 + \left(\frac{25}{4} - \frac{25}{2}\right)^2}$ Dist EC = $\frac{25\sqrt{5}}{8}$	
Area $\Delta EOC = \frac{1}{2} \left( 12\frac{1}{2} \right) \left( \frac{25\sqrt{5}}{8} \right) \sin 26,6^{\circ}$ Area $\Delta EOC = 19,555$	
Area of Trap = 31,2881+19,555 $\approx 50,8 \text{ units}^2$	

# Total: 150 mark